# Pre-AP Algebra 2 Instructional Planning Guide

Teacher Sample

The goal of the instructional planning guide is to help you create a roadmap of the key instructional activities and assessments   
you will use to design your course in alignment with the Pre-AP course framework and instructional principles. This sample   
illustrates one way in which you might use the guide. Pre-AP National Faculty and educators with experience teaching Pre-AP   
provided ideas for additional activities and resources that they might use alongside Pre-AP model lessons and formative   
assessment to build their full course.

**Using and Customizing Your Own Instructional Planning Guide:**

* When planning additional lessons, consider how they support the Pre-AP course framework, areas of focus,   
  and shared principles. These three elements represent the key ingredients of aligning to Pre-AP.
* Take time to capture your reflections as you move through the course.

## Unit 1: Modeling with Functions

| **Pacing in Min.** | **Date(s)** | **Key Concepts** | **Materials/Resources/Tasks**  *Pre-AP Model Lessons, Additional Lessons, Textbooks,  Performance Tasks, Assessments* | **Learning Objectives** | **State Standards** | **Reflections on  Areas of Focus & Shared Principles** |
| --- | --- | --- | --- | --- | --- | --- |
| ~90 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.1:  Recognizing Linear, Quadratic, and Exponential Relationships | 1.1.1 | A.SSE.1  F.IF.4 |  |
| ~60 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.2:  Making Predictions with Linear Models | 1.1.3, 1.1.4 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.5  F.IF.7  F.BF.1 |  |
| ~90 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.3:  Evaluating the Appropriateness of a Linear Model | 1.1.2, 1.1.3 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.7  F.BF.1 |  |
| ~90 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.4:  Connecting Growth Factor to Percent Change | 1.1.3, 1.1.4 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.5  F.IF.7  F.BF.1 |  |
| ~75 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.5:  Modeling Nonlinear Data with Exponential Functions | 1.1.3, 1.1.4 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.5  F.IF.7  F.BF.1 |  |
| ~45 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.6:  Modeling Scenarios with Quadratic Functions | 1.1.1, 1.1.3 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.7  F.BF.1 |  |
| ~60 |  | 1.1: Choosing Appropriate Function Models | Pre-AP Model Lesson 1.7:  Modeling Data with Quadratic Functions in Vertex Form | 1.1.1, 1.1.3 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.7  F.BF.1 |  |
| ~135 |  | 1.1: Choosing Appropriate Function Models | Practice Performance Task:  Modeling Bee Colony Collapse | 1.1.1, 1.1.2, 1.1.3, 1.1.4 | A.SSE.1  A.SSE.2  F.IF.4  F.IF.5  F.IF.7  F.BF.1 |  |
| ~45 |  | 1.1 | **Learning Checkpoint 1**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
| ~45 |  | 1.2: Rate of Change | Pre-AP Model Lesson 1.8:  Understanding Rate of Change | 1.2.1 | F.IF.4  F.IF.6 |  |
| ~60 |  | 1.2: Rate of Change | Pre-AP Model Lesson 1.9:  Average Rate of Change | 1.2.1, 1.2.2 | F.IF.4  F.IF.6 |  |
| ~45 |  | 1.2 | **Performance Task**  Counting Customers in the Grocery Store  *This performance task assesses learning objectives addressed in the unit.* | 1.2.1, 1.2.2 | F.IF.4  F.IF.6 |  |
| ~45 |  | 1.3: Piecewise-Defined Models | Pre-AP Model Lesson 1.10:  Modeling with Piecewise-Defined Functions | 1.3.1, 1.3.2 | A.CED.3  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1 |  |
| ~45 |  | 1.3: Piecewise-Defined Models | Graphing the Absolute Value Function as a Piecewise Function  Review with students how to solve absolute value equations. If *f* is an absolute value function, then the equationf open parentheses x close parentheses equals 0can have zero, one, or two solutions. The values of *x* that are solutions to the equation f open parentheses x close parentheses equals 0are the *x*-coordinates of the *x*-intercepts of the graph of y equals f open parentheses x close parentheses.  Use [**Desmos.com**](http://www.desmos.com) to have students practice breaking absolute value graphs into pairs of linear graphs. Students should recognize that the *x*-value of the vertex of the graph denotes the domains on which the pieces of the graph are defined. Be sure to include several different additive and multiplicative transformations of the function as a preview of Key Concept 2.2.  Debrief the lesson by asking questions about:   1. how to determine where the vertex of the graph occurs relative to the pieces 2. how to describe the domain and range of a piecewise function 3. why the domains of the pieces cannot overlap at the vertex   Students should start becoming familiar with both interval and inequality notation. | 1.3.3 | F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1 |  |
| ~45 |  | 1.2 and 1.3 | **Learning Checkpoint 2**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |

[add or remove rows as needed]

### Reflections

What went well in this unit?

When were students most engaged during this unit?

How have students grown? What opportunities for growth stand out at this time?

What needs modification or differentiation next time?

## Unit 2: The Algebra of Functions

| **Pacing in Min.** | **Date(s)** | **Key Concepts** | **Materials/Resources/Tasks**  *Pre-AP Model Lessons, Additional Lessons, Labs, Textbooks,  Performance Tasks, Assessments* | **Learning Objectives** | **State Standards** | **Reflections on  Areas of Focus & Shared Principles** |
| --- | --- | --- | --- | --- | --- | --- |
| ~110 |  | 2.1: Composing Functions | Pre-AP Model Lesson 2.1:  Introduction to Function Composition | 2.1.1, 2.1.2, 2.1.3 | A.CED.4  F.IF.4  F.IF.5  F.IF.8  F.BF.1 |  |
| ~45 |  | 2.1 and 1.3 | Pre-AP Model Lesson 2.2:  Function Composition with the Absolute Value Function | 1.3.3, 2.1.2 | A.CED.4  F.IF.4  F.IF.5  F.IF.8  F.BF.1 |  |
| ~60 |  | 2.2: Transforming Functions | Vertical and Horizontal Translations Using an Additive Transformation of *f*  Given a function *f* and a functiong open parentheses x close parentheses equals x plus k, graph three functions y equals f open parentheses x close parentheses, y equals f ring operator g open parentheses x close parentheses, and y equals g ring operator f open parentheses x close parentheses. Have students compare the graphs to observe that composition is not a commutative operation; specifically, f ring operator g open parentheses x close parentheses equals f open parentheses x plus k close parentheses is a horizontal translation, while g ring operator f open parentheses x close parentheses equals f open parentheses x close parentheses plus k is a vertical translation.  Remind students that they investigated translations in geometry and can apply their prior knowledge to construct graphs of translated functions. For example, if f open parentheses x close parentheses equals x squared, we can describe each point on the associated parabola in terms of a coordinate *p* and a corresponding coordinate p squared, represented by the coordinate pair open parentheses p comma p squared close parentheses. If the transformation of this function is a translation 3 units to the right, then the coordinates of the new point, the image of the point under the transformation, is open parentheses p minus 3 comma space open parentheses p minus 3 close parentheses squared close parentheses and the transformed function can be expressed algebraically as g open parentheses x close parentheses equals open parentheses x minus 3 close parentheses squared. If the transformation is a translation 2 units down, then the coordinates of the resulting point, the image of the point under the transformation, are open parentheses p comma p squared minus 2 close parentheses and the transformed function can be expressed algebraically as h open parentheses x close parentheses equals x squared minus 2.  Debrief this activity by leading a discussion about the patterns that students notice between vertical and horizontal translations. Ask students to create their own function transformations for other students to interpret and graph. | 2.2.1 | F.BF.3 |  |
| ~45 |  | 2.2: Transforming Functions | Vertical and Horizontal Dilations Using a Multiplicative Transformation of *f*  Given a function *f* and a function g open parentheses x close parentheses equals k x, where k not equal to 0, graph three functions y equals f open parentheses x close parentheses, y equals f ring operator g open parentheses x close parentheses, and y equals g ring operator f open parentheses x close parentheses. Have students observe that composition is not a commutative operation; specifically, f ring operator g open parentheses x close parentheses equals f open parentheses k x close parentheses is a horizontal dilation, while g ring operator f open parentheses x close parentheses equals k times f open parentheses x close parentheses is a vertical dilation.  Remind students that they investigated dilations in geometry and can apply their prior knowledge to construct graphs of dilated functions. For example, if f open parentheses x close parentheses equals x squared, we can describe each point on the associated parabola in terms of a coordinate *p* and a corresponding coordinate p squared, represented by the coordinate pair open parentheses p comma p squared close parentheses. If the transformation of this function is a horizontal dilation by a scale factor of 3, then the coordinates of the new point, the image of the point under the transformation, are open parentheses p over 3 comma p squared close parentheses and the transformed function can be expressed algebraically as g open parentheses x close parentheses equals open parentheses x over 3 close parentheses squared. If the transformation is a vertical dilation by a scale factor of 1 half, then the coordinates of the resulting point, the image of the point under the transformation, are open parentheses p comma open parentheses 1 half close parentheses p squared close parentheses and the transformed function can be expressed algebraically as h open parentheses x close parentheses equals open parentheses 1 half close parentheses x squared.  Debrief this activity by leading a discussion around the patterns that students notice between vertical and horizontal dilations. Have students practice using compositions to convert units of measure of an input or output quantity. For example, students can convert a function, *f*, whose output is expressed in feet, to a function whose output is expressed in inches using the dilation 12 times f. Ask students to create and explain their own transformations that can convert between different units of measure. | 2.2.2 | F.BF.3 |  |
| ~45 |  | 2.2: Transforming Functions | Function Transformations  In this lesson students practice performing transformations and engage in a discussion about the order of the transformations.  For example, define a function *f*, and then give students two functions such as g open parentheses x close parentheses equals negative f open parentheses x close parentheses and h open parentheses x close parentheses equals f open parentheses x close parentheses minus 1. Ask them to plot the three functions in the coordinate plane. Then define the composite function h ring operator g open parentheses x close parentheses equals negative f open parentheses x close parentheses minus 1 and ask the students if the order in which the transformations are applied matters. In other words, ask students if performing the transformations in different orders results in different output values. Students should see that the order of operations affects the order of the transformations.  Debrief this activity by having students practice different combinations of function transformations and observing how the results differ. | 2.2.3 | F.BF.1  F.BF.3 |  |
| ~45 |  | 2.2: Transforming Functions | Practice Performance Task:  Using Transformations to Model a Lion's Location | 2.2.1, 2.2.2, 2.2.3 | F.BF.1  F.BF.3 |  |
| ~45 |  | 2.1 and 2.2 | **Learning Checkpoint 1**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
| ~90 |  | 2.3: Inverting Functions | Pre-AP Model Lesson 2.3:  Inverting Operations | 2.3.1, 2.3.2 | F.IF.5  F.LE.4 |  |
| ~45 |  | 2.3: Inverting Functions | Making a Function Invertible by Restricting its Domain  Start the lesson by showing students an input-output table for a given function. Review that if a function *f* has an inverse, f to the power of negative 1 end exponent, then f open parentheses x close parentheses equals b implies f to the power of negative 1 end exponent open parentheses b close parentheses equals a.  Have students practice graphing functions and their inverses. Students may choose to plot inverses by switching the *x-* and *y*-values of coordinates of points on the graph or, equivalently, reflecting the points across the line y equals x. Be sure to include an example of a function whose domain must be restricted so it is invertible.  Debrief the activity by leading a discussion about the relationships between the domains and ranges of a function and its inverse. Be sure to elicit that the domain of a function is the range of its inverse function and the range of a function is the domain of its inverse function.  Provide students with a function, such as f open parentheses x close parentheses equals x squared plus 4, and ask if it is invertible. When they conclude that it is not, ask them how they could restrict its domain to make it invertible. You can challenge students to determine multiple ways to restrict the domain of a function so that it is invertible. | 2.3.3 | F.BF.4 |  |
| ~135 |  | 2.3: Inverting Functions | Pre-AP Model Lesson 2.4:  Introduction to Inverse Functions | 2.3.4, 2.3.5 | F.IF.4  F.BF.4 |  |
| ~45 |  | 2.3 | **Learning Checkpoint 2**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
| ~45 |  | 2.1 and 2.3 | **Performance Task**  Composite Functions and Inverse Functions  *This performance task assesses learning objectives addressed in the unit.* | 2.1.2, 2.1.3, 2.3.4 | A.CED.4  F.IF.4  F.IF.5  F.IF.8  F.BF.1  F.BF.4 |  |

[add or remove rows as needed]

### Reflections

What went well in this unit?

When were students most engaged during this unit?

How have students grown? What opportunities for growth stand out at this time?

What needs modification or differentiation next time?

## Unit 3: Function Families

| **Pacing in Min.** | **Date(s)** | **Key Concepts** | **Materials/Resources/Tasks**  *Pre-AP Model Lessons, Additional Lessons, Labs, Textbooks,  Performance Tasks, Assessments* | **Learning Objectives** | **State Standards** | **Reflections on  Areas of Focus & Shared Principles** |
| --- | --- | --- | --- | --- | --- | --- |
| ~60 |  | 3.1: Exponential and Logarithmic Functions | Pre-AP Model Lesson 3.1:  Problem Set for Exponential Functions | 3.1.1, 3.1.2 | A.CED.3  A.CED.4  F.IF.7  F.IF.8  F.BF.1 |  |
| ~125 |  | 3.1: Exponential and Logarithmic Functions | Pre-AP Model Lesson 3.2:  Introduction to the Logarithm Function | 3.1.3, 3.1.4, 3.1.5 | A.CED.3  A.CED.4  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1  F.BF.4  F.LE.4 |  |
| ~150 |  | 3.1: Exponential and Logarithmic Functions | Pre-AP Model Lesson 3.3:  Connecting Properties of Logarithms with Transformations of the Graph of the Parent Logarithm Function | 3.1.3, 3.1.4 | A.CED.3  A.CED.4  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1  F.LE.4 |  |
| ~90 |  | 3.1: Exponential and Logarithmic Functions | Pre-AP Model Lesson 3.4:  Applications of Logarithmic Functions | 3.1.4, 3.1.6 | F.IF.7  F.IF.8  F.LE.4 |  |
| ~45 |  | 3.1 and 1.1 | Practice Performance Task:  Modeling the Relationship Between Pressure and Volume | 1.1.1, 1.1.3, 3.1.3, 3.1.4 | A.CED.1  A.CED.2  F.IF.4  F.IF.8  F.BF.1  F.LE.4 |  |
| ~45 |  | 3.1 | **Learning Checkpoint 1**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
| ~45 |  | 3.2: Polynomial and Rational Functions | Pre-AP Model Lesson 3.5:  A Field Guide to Polynomial Functions | 3.2.1, 3.2.2, 3.2.3 | N.CN.7  N.CN.8  N.CN.9  A.SSE.1  A.SSE.2  A.APR.3  A.CED.2  A.CED.4  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1 |  |
|  |  | 3.2: Polynomial and Rational Functions | Perform Arithmetic with Complex Numbers  Review with students that the set of real numbers is a subset of the set of complex numbers, because every real number can be expressed as a plus 0 i. Have students practice adding, subtracting, and multiplying complex numbers.  After developing a basic understanding of operations with complex numbers, ask students to provide an equivalent expression for a given complex number, such as 3 plus 2 i. Examples might include: open parentheses 4 plus i close parentheses minus open parentheses 1 minus i close parentheses or open parentheses 1 half close parentheses left parenthesis 6 space plus space 4 i right parenthesis. After the class understands the task, divide students into groups of three or four and have them perform this equivalent representation activity with a different complex number. To make this activity more engaging, have each group use only one sheet of paper, with each student writing an equivalent expression for the complex number they are given and then passing the paper to the next student in their group. This writing and passing can continue within the group for 3 minutes. After time expires, each group will then share their answers in a round-robin format. You can award one point to a group for each correct response. The group with the most points at the end wins the game.  Debrief this activity by leading a discussion about imaginary solutions to quadratic equations and the use of nonreal zeros to create factors of a quadratic function. These factors can be multiplied to generate the general form of a quadratic function. | 3.2.4 | N.CN.1  N.CN.2  N.CN.7  N.CN.8  N.CN.9 |  |
|  |  | 3.2: Polynomial and Rational Functions | Constructing Representations of Rational Functions  Have students create several graphs using data sets that exhibit inverse relationships. Examples might include finding the time it takes to ascend a climbing wall given its height and the climber’s speed, finding the cost per student on a field trip given the total cost, calculating concentrations of mixtures for a given volume of solution, or calculating brightness given luminosity and surface area.  Debrief the lesson by having students make comparisons among the graphs and discussing common characteristics including domain and range of the functions. | 3.2.5 | A.SSE.1  A.SSE.2  A.CED.2  A.CED.4  F.IF.4  F.IF.8  F.BF.1 |  |
|  |  | 3.2: Polynomial and Rational Functions | Identifying Key Features of Graphs of Rational Functions  Have students discuss domain restrictions inherent in rational functions due to zeros of the polynomial in the denominator, and how these correspond to vertical asymptotes. Remember that “holes” in the graph are beyond the scope of this course, so you do not have to include them. Also discuss that the *x*-intercepts of the function correspond to the zeros in the numerator. Have students explore the end behavior of rational functions by analyzing the output values of the function for very large positive and negative values of the input.  After reviewing, divide the class into groups of four students. Each group should be given a rational function and must (a) identify the *x*- and *y*-intercepts, (b) identify the domain and range, (c) identify any vertical asymptotes, (d) graph the functions, and (e) describe the end behavior of the graph.  Debrief the lesson by discussing how key features of a rational function can be used to anticipate what the graph of the function looks like before using graphing technology, such as **Desmos.com**. | 3.2.6 | F.IF.4  F.IF.5  F.IF.8 |  |
| ~45 |  | 3.2 | **Performance Task**  Predicting the Number of Sections of a Circle  *This performance task assesses learning objectives addressed in the unit.* | 3.2.1 | A.SSE.1  A.CED.2  F.IF.4  F.IF.5  F.BF.1 |  |
|  |  | 3.3: Square Root and Cube Root Functions | Graphing Square Root Functions  Give students three or four square root functions along with their inverse functions. Students should sketch each graph along with its inverse, make observations about its domain, range, *x*-intercepts, *y*-intercepts, and end behavior. Be sure to have students explore why the domain of a square root function is often restricted.  Follow up by modeling some scenarios that involve motion. For example, the relationship between the time an object takes to fall to the ground and the height from which the object is dropped can be expressed as a square root function.  Debrief students’ answers to ensure they understand concepts related to solving square root equations and the connection between algebraic and graphical solutions. | 3.3.1  3.3.3 | A.SSE.1  A.CED.2  A.CED.4  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1  F.BF4 |  |
|  |  | 3.3: Square Root and Cube Root Functions | Graphing Cube Root Functions  The teacher-created activity [Graphing Cube Root Functions](https://teacher.desmos.com/activitybuilder/custom/5855ebc7a07418ea1c72d35e), available through Desmos Classroom Activities, builds on students’ understanding of square root functions. In this activity, students identify transformations to the parent cube root function, graph functions that meet specified characteristics, describe the transformations of a particular function from the parent function, and describe transformations of a given cube root function.  Follow up by modeling some scenarios that involve volume. For example, the relationship between the volume of a sphere and the radius of the sphere can be modeled by the inverse of the formula for the volume of a sphere Discuss the possible radii generated given differing volumes.  Debrief the lesson by discussing how the domain and range of a cube root function are the same as, and different from, that of a square root function. | 3.3.2 | A.SSE.1  A.CED.2  A.CED.4  F.IF.4  F.IF.5  F.IF.7  F.IF.8  F.BF.1 |  |
|  |  | 3.3: Square Root and Cube Root Functions | Solving Square Root and Cube Root Functions  Provide students with several scenarios involving square and cube root equations and have them determine solutions within the contextual scenario. For example, students could be asked to determine the hypotenuse of a right triangle given side lengths of 8 and *x*. An example of a scenario involving a cubic function might be calculating the age of an elephant, given that the shoulder heightof a male Asian elephant can be modeled by the function h left parenthesis t right parenthesis equals 62.5 left parenthesis t right parenthesis to the power of open parentheses 1 third close parentheses end exponent plus 75.8, where *t* is the age (in years) of the elephant and h left parenthesis t right parenthesis is the height of the elephant in centimeters.  Debrief the lesson by discussing how some square root functions have extraneous solutions and how contexts can be used to determine the reasonableness of the solutions. | 3.3.4 | A.REI.2  A.REI.11  A.CED.1 |  |
| ~45 |  | 3.2 and 3.3 | **Learning Checkpoint 2**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |

[add or remove rows as needed]

### Reflections

What went well in this unit?

When were students most engaged during this unit?

How have students grown? What opportunities for growth stand out at this time?

What needs modification or differentiation next time?

## Unit 4T: Trigonometric Functions

| **Pacing in Min.** | **Date(s)** | **Key Concepts** | **Materials/Resources/Tasks**  *Pre-AP Model Lessons, Additional Lessons, Labs, Textbooks,  Performance Tasks, Assessments* | **Learning Objectives** | **State Standards** | **Reflections on  Areas of Focus & Shared Principles** |
| --- | --- | --- | --- | --- | --- | --- |
| ~165 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.1:  Measuring an Angle's Openness | 4T.1.1 | F.TF.1 |  |
| ~45 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.2:  Determining Equivalent Angle Measures | 4T.1.1, 4T.1.2 | F.TF.1  F.TF.2 |  |
| ~45 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.3:  Angles in the Coordinate Plane | 4T.1.2 | F.TF.2 |  |
| ~90 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.4:  A Model for Circular Motion | 4T.1.3 | F.TF.2  A.CED.2  F.IF.5  F.IF.8  F.BF.1 |  |
| ~60 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.5:  The Coordinates of Points on a Circle | 4T.1.4 | F.TF.2 |  |
| ~75 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.6:  Common Reference Triangles in a Unit Circle | 4T.1.4, 4T.1.7 | F.TF.2  F.TF.8  A.CED.1 |  |
| ~120 |  | 4T.1: Radian Measure and Sinusoidal Functions | Pre-AP Model Lesson 4T.7:  A Model for Periodic Phenomena | 4T.1.5, 4T.1.6 | F.TF.5  A.CED.2  F.IF.4  F.IF.8  F.BF.1  F.BF.3 |  |
| ~45 |  | 4T.1 | **Performance Task**  Modeling Hours of Sunlight with a Trigonometric Function  *This performance task assesses learning objectives addressed in the unit.* | 4T.1.3, 4T.1.5 | F.TF.2  F.TF.5  A.CED.2  F.IF.4  F.IF.5  F.IF.7  F.BF.1  F.BF.3 |  |
| ~45 |  | 4T.1 | **Learning Checkpoint 1**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
| ~120 |  | 4T.2: The Tangent Function and Other Trigonometric Functions | Pre-AP Model Lesson 4T.8:  The Tangent Function | 4T.2.1, 4T.2.2 | F.TF.2  F.TF.5  A.CED.2  F.IF.7  F.BF.1 |  |
| ~45 |  | 4T.2: The Tangent Function and Other Trigonometric Functions | Practice Performance Task:  Connecting Circles, Triangles, and Line Segments | 4T.2.1, 4T.2.2 | F.TF.2  F.TF.5  A.CED.2  F.IF.7  F.BF.1 |  |
| ~60 |  | 4T.3: Inverting Trigonometric Functions | Inverting Trigonometric Functions  Have students discuss their prior experience with domain restrictions. In Units 2 and 3, students explored how to restrict the domain of a function so that it is invertible. This lesson builds on that understanding.  Students are familiar with graphing trigonometric functions from earlier lessons in the unit and should use their graphs to help with the discussion. Trigonometric functions are periodic, so each output value is associated with more than one input value. For a trigonometric function to have an inverse function, the domain of each function must be restricted so each output value is associated with exactly one input value. Such a relationship is described as *one-to-one*. Have students determine a reasonable domain for each of the trigonometric functions–sine, cosine, and tangent–so each has an inverse. Then they can share their initial responses with a partner to discuss how they might best restrict the domain.  Debrief the lesson by having several pairs of students share their domain restrictions and rationales. Let students know that inverse trigonometric functions often use the prefix *arc*- because the output of an inverse trigonometric function is an arc length measured in radians. Through this discussion let students know that sine, cosine, and tangent functions usually require domain restrictions to be invertible, but that many such domain restrictions are possible. | 4T.3.1 | F.BF.4 |  |
| ~90 |  | 4T.3: Inverting Trigonometric Functions | Solving Equations Involving Trigonometric Functions  This lesson builds on the previous lesson in which students restrict the domain of trigonometric functions so that they are invertible. You can open the lesson by asking students to describe the difference between solving the equation cos left parenthesis x right parenthesis space equals fraction numerator space 1 over denominator 2 end fraction and evaluating the expression text arc end text cos open parentheses 1 half close parentheses. Elicit that there are infinitely many solutions for cos left parenthesis x right parenthesis space equals fraction numerator space 1 over denominator 2 end fraction but that there is only one solution for text arc end text cos open parentheses 1 half close parentheses. Students may benefit from contemplating the difference between x squared space equals space 9 and square root of 9. Note that solving a trigonometric equation for all possible solutions is beyond the scope of course. Students only need to find solutions over a finite interval, often one that is suggested by a contextual scenario.  Ask students to provide some real-world examples of periodic phenomena. Their responses might include the height of a rotating Ferris wheel over time or the rise and fall of an animal population over seasons of the year. Have students answer questions about such scenarios involving both inputs and outputs over a given interval.  Debrief the lesson by leading a discussion about the number of solutions that students should determine for a trigonometric function over a finite interval. Students can use the graph of a trigonometric function to identify that the number of solutions depends on the interval of interest. | 4T.3.2 | A.REI.11  A.CED.1 |  |
| ~45 |  | 4T.2 and 4T.3 | **Learning Checkpoint 2**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |

[add or remove rows as needed]

### Reflections

What went well in this unit?

When were students most engaged during this unit?

How have students grown? What opportunities for growth stand out at this time?

What needs modification or differentiation next time?

## Unit 4M: Matrices and Their Applications

| **Planned Date(s)** | **Actual Date(s)** | **Key Concepts** | **Materials/Resources/Tasks**  *Pre-AP Model Lessons, Additional Lessons, Labs, Textbooks,  Performance Tasks, Assessments* | **Learning Objectives** | **State Standards** | **Reflections on  Areas of Focus & Shared Principles** |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.1:  Introduction to Linear Transformations | 4M.1.1 | F.IF.8 |  |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.2:  Expressing Linear Transformations with Matrix Multiplication | 4M.1.2, 4M.1.3 | A.CED.3  F.BF.1 |  |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.3:  Determining Images of Multiple Points Simultaneously | 4M.1.3, 4M.1.4 | A.APR.1 |  |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.4:  Area and the Determinant of a Matrix | 4M.1.5 |  |  |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.5:  Sequences of Linear Transformations | 4M.1.6 | F.BF.3 |  |
|  |  | 4M.1: Geometric Transformations | Pre-AP Model Lesson 4M.6:  Undoing Transformations and Finding Preimages | 4M.1.7, 4M.1.8 | F.BF.4 |  |
|  |  | 4M.1: Geometric Transformations | **Learning Checkpoint 1**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
|  |  | 4M.2: Solving Systems of Equations with Matrices | Representing Systems of Linear Equations Using Matrices  This lesson should start with a very brief review of systems of linear equations and solving them using either the substitution or elimination method. Restrict the review to system of linear equation written in standard form. This will be important when building understanding of a system of linear equations written as a matrix equation. You can have students solve the following system of linear equations or choose your own:    After students solve the system, explain that they will explore another representation of a system of linear equations. To start the exploration, have students write what they notice and wonder as they compare and contrast the representations below:    Next, have students share what they notice and wonder and then have them multiply the matrices on the left side of the matrix equation. Help them see that the algebraic form of the linear system equation is an equivalent form of the matrix equation representation.  Students should practice moving between systems of equations and matrix equations, understanding that they may need to rewrite the system in a particular form so that it can be represented as a matrix equation. | 4M.2.1 | A.CED.2  A.CED.3 |  |
|  |  | 4M.2: Solving Systems of Equations with Matrices | Solving Systems Using Matrices  Begin the lesson by having students think about how to solve the systems of equations represented as a matrix equation. The matrix equation presented in the lesson above follows:    Give students some time to discuss how they would use a matrix equation to solve the system of equations. Have students share their ideas with the class. Some students may recognize that they can multiply each side of the matrix equation by the inverse of the coefficient matrix to isolate . Students have prior experiences with finding the inverse of a matrix and using the inverse to determine the preimage of a given point to support this thinking.  Progress students to the general form of a matrix equation . Be sure to define *A*, *X* and *B*. Ask students to discuss with their partner/group how they would go about solving this matrix equation. Help them recall that and  are both equal to the identity matrix. Then solve the matrix equation.  For the next part of the lesson, help students understand how useful matrices are when solving systems of equations. Give students a system of 3 linear equations with 3 unknowns and attempt to solve the system using one of the reviewed algebraic methods. Note the tedium and length of producing such a solution and indicate the need for another method.  To necessitate the need for another method to produce the solution to a system, have students, without technology, multiply a  matrix of coefficients for the given system of equations and the matrix consisting of the system’s unknowns. Students should note that the product is precisely one side of the system of equations. Observe that any system of linear equations can be written as the matrix equation where *A* is the  matrix consisting of the system’s coefficients, *X*  is the matrix consisting of the system’s unknown variables, and *B* is thematrix consisting of the constants that each respective equation is equal to.  Students should be provided practice with both contextualized and decontextualized systems of equations where they can write these systems as matrix equations and use technology to solve for *X* and thereby solve the system. For the systems given in context, an emphasis should be put on interpreting the solutions to the matrix equations in context of the problem. When solving a system using students should recall that  exists only if its determinant is nonzero. In the case that an inverse does not exist (the determinant of A is equal to 0), students may need to use other methods (a graphical approach) to determine whether there is a solution (nonunique). | 4M.2.2 | A.CED.3 |  |
|  |  | 4M.2: Solving Systems of Equations with Matrices | Using Matrices to Write Polynomial Functions  In the same way that we can use 2 points to determine the algebraic representation of a line, systems of equations can be used to determine the algebraic representation of a polynomial of degree *n* if given points that lie on the graph of that polynomial. In this case, the coefficients of each term of the polynomial become your unknowns and the function can be evaluated at each of the points to build a system of linear equations whose solutions represent each coefficient in the polynomial.  Students should be given arbitrary polynomials that satisfy points that can be turned into a system of equations whose solutions yield the coefficients in the algebraic representation of the polynomial. For example, the quadratic function of the form passes through the points , , and . This implies that ,and . This yields a system of 3 equations with unknowns a, b, and c which can then be represented as a matrix equation where *A* is a matrix*, X* is the  matrix with entries *a*, *b*, and *c*, and *B* is the matrix with entries 3, -5, and 3. The solution to the system yields  implying that .  Debrief this activity by providing students additional practice on constructing polynomial functions using matrices. Encourage students to use their graphing calculators to test the accuracy of the polynomials they constructed using matrices. | 4M.2.3 |  |  |
|  |  | 4M.2: Solving Systems of Equations with Matrices | Practice Performance Task:  Using Matrices to Construct Polynomial Functions | 4M.2.1, 4M.2.2, 4M.2.3 | A.CED.2  A.CED.3 |  |
|  |  | 4M.3: Applications of Matrix Multiplication | Pre-AP Model Lesson 4M.7:  Introduction to Recursive Processes | 4M.3.1, 4M.3.2 | A.CED.2  F.IF.8 |  |
|  |  | 4M.3: Applications of Matrix Multiplication | Pre-AP Model Lesson 4M.8:  Stabilized Recursive Processes | 4M.3.2, 4M.3.3 | A.CED.2  F.IF.8 |  |
|  |  | 4M.3: Applications of Matrix Multiplication | Determining the Previous Steps in a Recursive Process  Students will have built an understanding of how things change over time within a recursive process. In this lesson, students will use current/future information of a recursive process to determine what happened previously. That is, given the matrix A describing how members of a population move between subpopulations, and the size of the populations in the *kth* time step , students will solve for the size of the subpopulations in the time step. This can be done by using the inverse matrix  to solve forin the matrix equation.  Debrief this activity by having students find the sizes of previous subpopulation at any time step. Practice problems should only have students determine the sizes of a subpopulation one previous time step away. Having students identify sizes of subpopulation by multiple previous time steps away should not be included in this lesson. | 4M.3.4 | A.CED.2  F.IF.8 |  |
|  |  | 4M.2 and 4M.3 | **Learning Checkpoint 2**  *This learning checkpoint can assess any of the learning objectives from its associated key concepts.* |  |  |  |
|  |  | 4M.3 | **Performance Task**  Migrating Populations  *This performance task assesses learning objectives addressed in the unit.* |  |  |  |

[add or remove rows as needed]

### Reflections

What went well in this unit?

When were students most engaged during this unit?

How have students grown? What opportunities for growth stand out at this time?

What needs modification or differentiation next time?