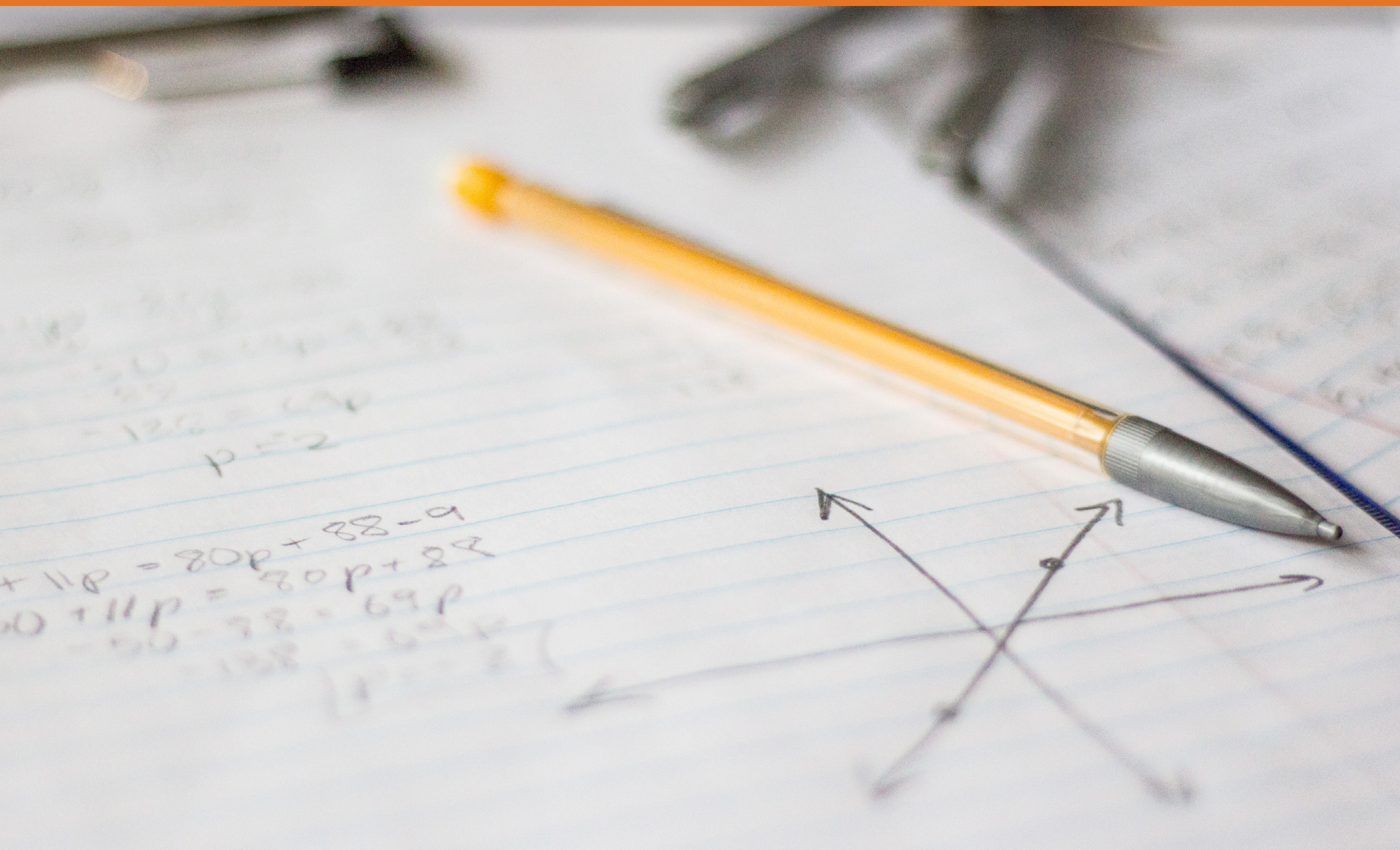
CollegeBoard Pre-AP®

Pre-AP® Algebra 1

COURSE GUIDE

INCLUDES

- Approach to teaching and learning
- Course map
- Course framework
- Sample assessment questions



CollegeBoard Pre-AP®

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COURSE GUIDE

Revised Fall 2026

Please visit Pre-AP online at **preap.org** for more information and updates about the course and program features.

ABOUT COLLEGE BOARD

College Board reaches more than 7 million students a year, helping them navigate the path from high school to college and career. Our not-for-profit membership organization was founded more than 120 years ago. We pioneered programs like the SAT® and AP® to expand opportunities for students and help them develop the skills they need. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

For further information, visit **www.collegeboard.org**.

ENROLLING STUDENTS: ACCESS, OPPORTUNITY, AND READINESS

The AP Program welcomes all students willing to challenge themselves with college-level coursework and career preparation. We strongly encourage educators to invite students into AP classes, including students from ethnic, racial, socioeconomic, geographic, or other groups not broadly participating in a school's AP program. We believe that readiness for AP is attainable, and that educators can expand readiness by opening access to Pre-AP course work. We commit to support educators and communities in their efforts to make AP courses widely available, advancing students in their plans for college and careers.

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Acknowledgments

College Board would like to acknowledge the following present and past committee members, reviewers, and College Board staff for their assistance with and commitment to the development of this course. All individuals and their affiliations were current at the time of their contribution.

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About Pre-AP

Introduction to Pre-AP

Every student deserves classroom opportunities to learn, grow, and succeed. College Board developed Pre-AP® to deliver on this simple premise. Pre-AP courses are designed to support all students across varying levels of readiness. They are not honors or advanced courses.

Participation in Pre-AP courses allows students to slow down and focus on the most essential and relevant concepts and skills. Students have frequent opportunities to engage deeply with texts, sources, and data as well as compelling higher-order questions and problems. Across Pre-AP courses, students experience shared instructional practices and routines that help them develop and strengthen the important critical thinking skills they will need to employ in high school, college, and life. Students and teachers can see progress and opportunities for growth through varied classroom assessments that provide clear and meaningful feedback at key checkpoints throughout each course.

DEVELOPING THE PRE-AP COURSES

Pre-AP courses are carefully developed in partnership with experienced educators, including middle school, high school, and college faculty. Pre-AP educator committees work closely with College Board to ensure that the course resources define, illustrate, and measure grade-level-appropriate learning in a clear, accessible, and engaging way. College Board also gathers feedback from a variety of stakeholders, including Pre-AP partner schools from across the nation who have participated in multiyear pilots of select courses. Data and feedback from partner schools, educator committees, and advisory panels are carefully considered to ensure that Pre-AP courses provide all students with grade-level-appropriate learning experiences that place them on a path to college and career readiness.

SUPPORTING AP READINESS

The following actions support student preparedness for AP courses:

1. **Pre-AP Courses for All:** Increase the number of students taking AP courses by having all students engage in the Pre-AP program.
2. **Course Frameworks:** Align instruction to Pre-AP course frameworks so students gain access to the content and skills necessary for AP success.
3. **Assessments:** Provide students practice with Pre-AP assessments to prepare them for the question types on AP Exams.

4. **Professional Learning:** Engage teachers, instructional leaders, and administrators in Pre-AP Professional Learning to gain tools for creating Pre-AP courses where all students develop skills for AP success and to collaborate with a supportive community of Pre-AP educators for ongoing support.

PRE-AP EDUCATOR NETWORK

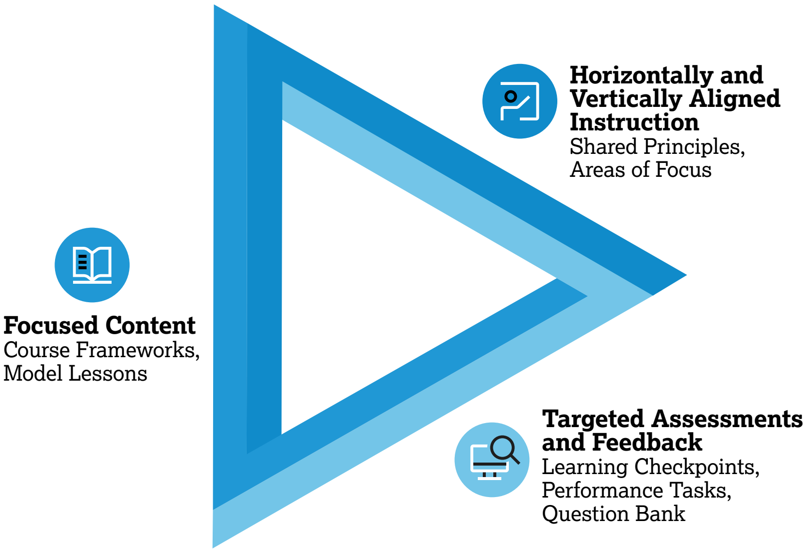
Similar to the way in which teachers of Advanced Placement® (AP®) courses can become more deeply involved in the program by becoming AP Readers or workshop consultants, Pre-AP teachers also have opportunities to become active in their educator network. Each year, College Board expands and strengthens the Pre-AP National Faculty—the team of educators who facilitate Pre-AP Professional Learning. Pre-AP teachers can also become curriculum and assessment contributors by working with College Board to design, review, or pilot the course resources.

HOW TO GET INVOLVED

Schools and districts interested in learning more about participating in Pre-AP should visit **preap.org** or contact us at **preap@collegeboard.org**.

Pre-AP Approach to Teaching and Learning

Pre-AP courses invite all students to learn, grow, and succeed through focused content, horizontally and vertically aligned instruction, and targeted assessments for learning. The Pre-AP approach to teaching and learning, as described below, is not overly complex, yet the combined strength results in powerful and lasting benefits for both teachers and students. This is our theory of action.



FOCUSED CONTENT

Pre-AP courses focus deeply on a limited number of concepts and skills with the broadest relevance for high school coursework and college and career success. The course framework serves as the foundation of the course and defines these prioritized concepts and skills. Pre-AP model lessons and assessments are based directly on this focused framework. The course design provides students and teachers with intentional permission to slow down and focus.

HORIZONTALLY AND VERTICALLY ALIGNED INSTRUCTION

Shared principles cut across all Pre-AP courses and disciplines. Each course is also aligned to discipline-specific areas of focus that prioritize the critical reasoning skills and practices central to that discipline.

SHARED PRINCIPLES

All Pre-AP courses share the following set of research-supported instructional principles. Classrooms that regularly focus on these cross-disciplinary principles allow students to effectively extend their content knowledge while strengthening their critical thinking skills. When students are enrolled in multiple Pre-AP courses, the horizontal alignment of the shared principles provides students and teachers across disciplines with a shared language for their learning and investigation, and multiple opportunities to practice and grow. The critical reasoning and problem-solving tools students develop through these shared principles are highly valued in college coursework and in the workplace.

Close Observation and Analysis

Students are provided time to carefully observe one data set, text, image, performance piece, or problem before being asked to explain, analyze, or evaluate. This creates a safe entry point to simply express what they notice and what they wonder. It also encourages students to slow down and capture relevant details with intentionality to support more meaningful analysis, rather than rush to completion at the expense of understanding.

Higher-Order Questioning

Students engage with questions designed to encourage thinking that is elevated beyond simple memorization and recall. Higher-order questions require students to make predictions, synthesize, evaluate, and compare. As students grapple with these questions, they learn that being inquisitive promotes extended thinking and leads to deeper understanding.

Evidence-Based Writing

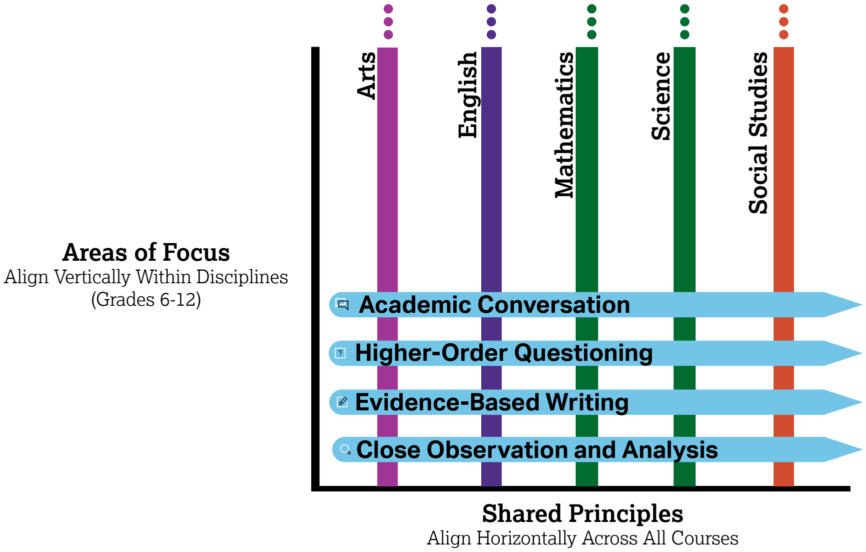
With strategic support, students frequently engage in writing coherent arguments from relevant and valid sources of evidence. Pre-AP courses embrace a purposeful and scaffolded approach to writing that begins with a focus on precise and effective sentences before progressing to longer forms of writing.

Academic Conversation

Through peer-to-peer dialogue, students' ideas are explored, challenged, and refined. As students engage in academic conversation, they come to see the value in being open to new ideas and modifying their own ideas based on new information. Students grow as they frequently practice this type of respectful dialogue and critique and learn to recognize that all voices, including their own, deserve to be heard.

AREAS OF FOCUS

The areas of focus are discipline-specific reasoning skills that students develop and leverage as they engage with content. Whereas the shared principles promote horizontal alignment across disciplines, the areas of focus provide vertical alignment within a discipline, giving students the opportunity to strengthen and deepen their work with these skills in subsequent courses in the same discipline.



For information about the Pre-AP mathematics areas of focus, see page 13.

TARGETED ASSESSMENTS FOR LEARNING

Pre-AP courses include strategically designed classroom assessments that serve as tools for understanding progress and identifying areas that need more support. The assessments provide frequent and meaningful feedback for both teachers and students across each unit of the course and for the course as a whole. For more information about assessments in Pre-AP Algebra 1, see page 54.

Pre-AP Professional Learning

To build teacher confidence and student preparedness for AP, it's best practice for teachers new to Pre-AP to engage in two professional learning opportunities:

1. The first is designed to help prepare teachers to teach their specific course. Teachers can choose from two options: the Pre-AP Summer Institute and the Online Foundational Modules. Both options provide continuing education units upon completion.
   - The Pre-AP Summer Institute provides a collaborative experience that empowers participants to prepare and plan for their Pre-AP course. While attending, teachers engage with Pre-AP course frameworks, shared principles, areas of focus, and sample model lessons. Participants are given supportive planning time where they work with peers to begin building their Pre-AP course plan.
   - Online Foundational Modules are available to all teachers of Pre-AP courses. In this 12- to 20-hour asynchronous course, teachers explore Pre-AP instructional resources, learn best practices for teaching Pre-AP, and begin to build their course.
2. The second helps teachers prepare for Pre-AP Performance Tasks. Online scoring modules offer guidance and practice for applying scoring guidelines and examining student work. Teachers may complete the modules independently or with teachers of the same course in their school's professional learning communities.

For more information about Pre-AP professional learning opportunities, visit **https://pre-ap.collegeboard.org/professional-learning**.

About Pre-AP Algebra 1

Introduction to Pre-AP Algebra 1

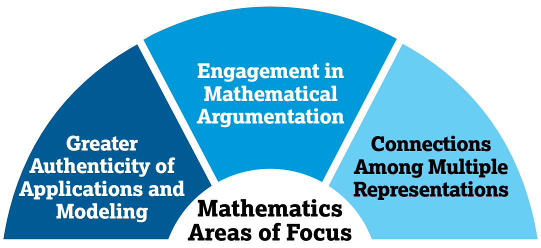
The Pre-AP Algebra 1 course is designed to deepen students' understanding of linear relationships by emphasizing patterns of change, multiple representations of functions and equations, modeling real-world scenarios with functions, and methods for finding and representing solutions of equations and inequalities. Taken together, these ideas provide a powerful set of conceptual tools that students can use to make sense of their world through mathematics.

Rather than seeking to cover all topics traditionally included in a standard algebra textbook, this course focuses on the foundational algebraic knowledge and skills that matter most for college and career readiness. The Pre-AP Algebra 1 Course Framework highlights how to guide students to connect core ideas within and across the units of the course, promoting a coherent understanding of linear relationships.

The components of this course have been crafted to prepare not only the next generation of mathematicians, scientists, programmers, statisticians, and engineers, but also a broader base of mathematically informed citizens who are well equipped to respond to the array of mathematics-related issues that impact our lives at personal, local, and global levels.

PRE-AP MATHEMATICS AREAS OF FOCUS

The Pre-AP mathematics areas of focus, shown below, are mathematical practices that students develop and leverage as they engage with content. They were identified through educator feedback and research about where students and teachers need the most curriculum support. These areas of focus are vertically aligned to the mathematical practices embedded in other mathematics courses in high school, including AP, and in college, giving students multiple opportunities to strengthen and deepen their work with these skills throughout their educational career. They also support and align to mathematical practices and skills in AP courses, including Calculus, Statistics, Precalculus, and Computer Science Principles, and to the mathematical practices listed in various state standards.



Greater Authenticity of Applications and Modeling

Students create and use mathematical models to understand and explain authentic scenarios.

Mathematical modeling is a process that helps people explore, represent, analyze, and explain the world. In Pre-AP Algebra 1, students explore real-world contexts where mathematics can be used to make sense of a situation. They engage in the modeling process by making choices about what aspects of the situation to model, assessing how well the model represents the available data, drawing conclusions from their model, justifying decisions they make through the process, and identifying what the model helps clarify and what it does not.

In addition to mathematical modeling, Pre-AP Algebra 1 students engage in mathematics through authentic applications. Applications are similar to modeling problems in that they are drawn from real-world phenomena, but they differ because the applications dictate the appropriate mathematics to use to solve the problem. Pre-AP Algebra 1 balances these two types of real-world tasks.

Engagement in Mathematical Argumentation

Students use evidence to craft mathematical conjectures and prove or disprove them.

Conjecture, reasoning, and proof lie at the heart of the discipline of mathematics. Mathematics is both a way of thinking and a set of tools for solving problems. Pre-AP Algebra 1 students gain experience, comfort, and proficiency with mathematical thinking by observing and generalizing patterns in number sequences, graphs, equations, operations, and functions. They harness their curiosity to create problems to solve and conjectures to prove or disprove. Through mathematical argumentation, students learn how to be critical of their own reasoning and the reasoning of others.

Connections Among Multiple Representations

Students represent mathematical concepts in a variety of forms and move fluently among the forms.

Mathematical concepts can be represented in a variety of forms. Pre-AP Algebra 1 students learn how the multiple representations of a concept are connected to each other and how to fluently translate between graphical, numerical, algebraic, and verbal representations. Every mathematical representation illuminates certain characteristics of a concept while also obscuring other aspects. With experience that begins to develop in Pre-AP Algebra 1, students develop a nuanced understanding of which representations best serve a particular purpose.

PRE-AP ALGEBRA 1 AND CAREER READINESS

The Pre-AP Algebra 1 course resources are designed to expose students to a wide range of career opportunities that depend upon Algebra 1 knowledge and skills. Examples include not only field-specific careers such as mathematician or statistician but also other endeavors where algebraic knowledge is relevant and applicable, such as actuaries, engineers, programmers, carpenters, and HVAC technicians.

Career clusters that involve mathematics, along with examples of careers in mathematics and other careers that require the use of algebra, are provided below. Teachers should consider discussing these with students throughout the year to promote motivation and engagement.

Career Clusters Involving Mathematics
arts, A/V technology, and communications architecture and construction business management and administration finance government and public administration health science information technology manufacturing marketing STEM (science, technology, engineering, and math)

Examples of Mathematics Careers	Examples of Algebra 1 Related Careers
actuary financial analyst mathematician mathematics teacher professor programmer statistician	carpenter computer programmer economist electrician engineer HVAC technician operations research analyst programmer

Source for Career Clusters: "Advanced Placement and Career and Technical Education: Working Together." Advance CTE and the College Board. October 2018.

For more information about careers that involve mathematics, teachers and students can visit and explore the College Board's Big Future resources: **https://bigfuture.collegeboard.org/**.

SUMMARY OF RESOURCES AND SUPPORTS

Teachers are strongly encouraged to take advantage of the full set of resources and supports for Pre-AP Algebra 1, which is summarized below.

COURSE FRAMEWORK

Included in this guide as well as in the *Pre-AP Algebra 1 Teacher Resources*, the framework defines what students should know and be able to do by the end of the course. It serves as an anchor for model lessons and assessments, and it is the primary resource needed to plan the course. **Aligning classroom instruction to the course framework is strongly recommended**. *For more details see page 20.*

MODEL LESSONS

Teacher resources, available in print and online, include a robust set of model lessons that demonstrate how to translate the course framework, shared principles, and areas of focus into daily instruction. **Use of the model lessons is recommended**. *For more details see page 52.*

LEARNING CHECKPOINTS

Accessed through Pre-AP Classroom (the Pre-AP digital platform), these short formative assessments provide insight into student progress. They are automatically scored and include multiple-choice and technology-enhanced items with rationales that explain correct and incorrect answers. **Use of learning checkpoints is recommended**. *For more details see page 54.*

PERFORMANCE TASKS

Available in the printed teacher resources as well as on Pre-AP Classroom, performance tasks allow students to demonstrate their learning through extended problem-solving, writing, analysis, and/or reasoning tasks. Scoring guidelines are provided to inform teacher scoring, with additional practice and feedback suggestions available in online modules on Pre-AP Classroom. **Use of performance tasks is recommended**. *For more details see page 56.*

PRACTICE PERFORMANCE TASKS

Available in the student resources, with supporting materials in the teacher resources, these tasks provide an opportunity for students to practice applying skills and knowledge as they would in a performance task, but in a more scaffolded environment. **Use of the practice performance tasks is recommended**. *For more details see page 56.*

QUESTION BANK

Accessed through Pre-AP Classroom, the question bank can be used to create assessments of various lengths such as exit tickets, quizzes, semester tests, and more. Teachers can author their own questions, create their own assessments, and share quizzes created with items from the question bank with other educators. *For more details see page 65.*

PROFESSIONAL LEARNING

College Board encourages Pre-AP teachers to attend a Pre-APSI (in person or virtually) or complete the Pre-AP Online Foundational Module Series to support their preparation and planning for teaching a Pre-AP course. Teachers also have access to Online Performance Task Scoring Modules, which provide practical guidance for administering and scoring performance tasks. *For more details see page 9.*

Course Map

PLAN

The course map shows how components are positioned throughout the course. As the map indicates, the course is designed to be taught over 140 class periods (based on 45-minute class periods), for a total of 28 weeks.

Model lessons are included for approximately 50% of the total instructional time, with the percentage varying by unit. Each unit is divided into key concepts.

TEACH

The model lessons demonstrate how the Pre-AP shared principles and mathematics areas of focus come to life in the classroom.

Shared Principles
Close observation and analysis
Higher-order questioning
Evidence-based writing
Academic conversation

Areas of Focus
Greater authenticity of applications and modeling
Engagement in mathematical argumentation
Connections among multiple representations

ASSESS AND REFLECT

Each unit includes two learning checkpoints and a performance task. These formative assessments are designed to provide meaningful feedback for both teachers and students.

UNIT 1 **Linear Functions and Equations**

~50 Class Periods

Pre-AP model lessons provided for approximately 56% of instructional time in this unit

KEY CONCEPT 1.1
Modeling with Functions That Have a Constant Rate of Change

KEY CONCEPT 1.2
Representing Linear Functions in Multiple Ways

KEY CONCEPT 1.3
Modeling Linear Relationships with Constraints

Learning Checkpoint 1

KEY CONCEPT 1.4
Solving Systems of Linear Equations and Linear Inequalities

Performance Task for Unit 1

Learning Checkpoint 2

UNIT 2 Exponential Functions and Expressions

~30 Class Periods

Pre-AP model lessons provided for approximately 56% of instructional time in this unit

KEY CONCEPT 2.1
Modeling with Functions That Have Multiplicative Growth or Decay

KEY CONCEPT 2.2
Interpreting Equivalent Exponential Expressions

Learning Checkpoint 1

KEY CONCEPT 2.3
Extending Multiplicative Patterns of Change

Learning Checkpoint 2

Performance Task for Unit 2

UNIT 3 Quadratic Functions and Equations

~40 Class Periods

Pre-AP model lessons provided for approximately 50% of instructional time in this unit

KEY CONCEPT 3.1
Modeling with Functions That Have a Linear Rate of Change

KEY CONCEPT 3.2
Representing Quadratic Functions and Equations in Multiple Ways

Learning Checkpoint 1

KEY CONCEPT 3.3
Solving Quadratic Equations

Learning Checkpoint 2

Performance Task for Unit 3

UNIT 4 Comparing and Selecting Functions

~20 Class Periods

Pre-AP model lessons provided for approximately 53% of instructional time in this unit

KEY CONCEPT 4.1
Comparing and Analyzing Functions

KEY CONCEPT 4.2
Using Functions to Model Contextual Scenarios

Learning Checkpoint 1

KEY CONCEPT 4.3
Using Functions to Model Data Sets

Learning Checkpoint 2

Performance Task for Unit 4

Pre-AP Algebra 1 Course Framework

INTRODUCTION

Based on the Understanding by Design® (Wiggins and McTighe) model, the Pre-AP Algebra 1 Course Framework is back mapped from AP expectations and aligned to essential grade-level expectations. The course framework serves as a teacher's blueprint for the Pre-AP Algebra 1 instructional resources and assessments.

The course framework was designed to meet the following criteria:

- **Focused:** The framework provides a deep focus on a limited number of concepts and skills that have the broadest relevance for later high school, college, and career success.
- **Measurable:** The framework's learning objectives are observable and measurable statements about the knowledge and skills students should develop in the course.
- **Manageable:** The framework is manageable for a full year of instruction, fosters the ability to explore concepts in depth, and enables room for additional local or state standards to be addressed where appropriate.
- **Accessible:** The framework's learning objectives are designed to provide all students, across varying levels of readiness, with opportunities to learn, grow, and succeed.

COURSE FRAMEWORK COMPONENTS

The Pre-AP Algebra 1 Course Framework includes the following components:

Big Ideas

The big ideas are recurring themes that allow students to create meaningful connections between course concepts. Revisiting the big ideas throughout the course and applying them in a variety of contexts allow students to develop deeper conceptual understandings.

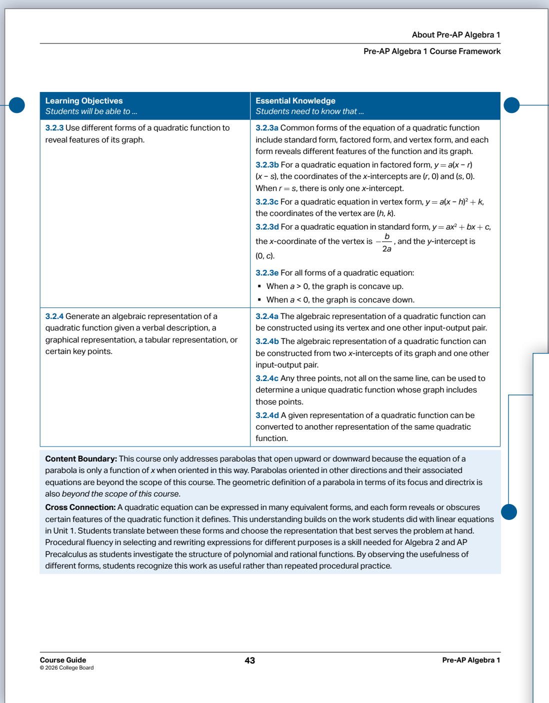
Enduring Understandings

Each unit focuses on a small set of enduring understandings. These are the long-term takeaways related to the big ideas that leave a lasting impression on students. Students build and earn these understandings over time by exploring and applying course content throughout the year.

Key Concepts

To support teacher planning and instruction, each unit is organized by key concepts. Each key concept includes relevant **learning objectives** and **essential knowledge statements** and may also include **content boundary and cross connection statements**. These are illustrated and defined below.

Learning Objectives:
These objectives define what a student needs to be able to do with essential knowledge to progress toward the enduring understandings. The learning objectives serve as actionable targets for instruction and assessment.



About Pre-AP Algebra 1

Pre-AP Algebra 1 Course Framework

Learning Objectives *Students will be able to ...*	Essential Knowledge *Students need to know that ...*
3.2.3 Use different forms of a quadratic function to reveal features of its graph.	3.2.3a Common forms of the equation of a quadratic function include standard form, factored form, and vertex form, and each form reveals different features of the function and its graph. 3.2.3b For a quadratic equation in factored form, $y = a(x - r)(x - s)$, the coordinates of the x-intercepts are $(r, 0)$ and $(s, 0)$. When $r = s$, there is only one x-intercept. 3.2.3c For a quadratic equation in vertex form, $y = a(x - h)^2 + k$, the coordinates of the vertex are (h, k). 3.2.3d For a quadratic equation in standard form, $y = ax^2 + bx + c$, the x-coordinate of the vertex is $-\frac{b}{2a}$, and the y-intercept is $(0, c)$. 3.2.3e For all forms of a quadratic equation: ▪ When $a > 0$, the graph is concave up. ▪ When $a < 0$, the graph is concave down.
3.2.4 Generate an algebraic representation of a quadratic function given a verbal description, a graphical representation, a tabular representation, or certain key points.	3.2.4a The algebraic representation of a quadratic function can be constructed using its vertex and one other input-output pair. 3.2.4b The algebraic representation of a quadratic function can be constructed from two x-intercepts of its graph and one other input-output pair. 3.2.4c Any three points, not all on the same line, can be used to determine a unique quadratic function whose graph includes those points. 3.2.4d A given representation of a quadratic function can be converted to another representation of the same quadratic function.

Content Boundary: This course only addresses parabolas that open upward or downward because the equation of a parabola is only a function of x when oriented in this way. Parabolas oriented in other directions and their associated equations are beyond the scope of this course. The geometric definition of a parabola in terms of its focus and directrix is also *beyond the scope of this course.*

Cross Connection: A quadratic equation can be expressed in many equivalent forms, and each form reveals or obscures certain features of the quadratic function it defines. This understanding builds on the work students did with linear equations in Unit 1. Students translate between these forms and choose the representation that best serves the problem at hand. Procedural fluency in selecting and rewriting expressions for different purposes is a skill needed for Algebra 2 and AP Precalculus as students investigate the structure of polynomial and rational functions. By observing the usefulness of different forms, students recognize this work as useful rather than repeated procedural practice.

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Essential Knowledge Statements:
Each essential knowledge statement is linked to a learning objective. One or more essential knowledge statements describe the knowledge required to perform each learning objective.

Content Boundary and Cross Connection Statements:
When needed, content boundary statements provide additional clarity about the content and skills that lie within versus outside of the scope of this course.

Cross connection statements highlight important connections that should be made between key concepts within and across the units.

BIG IDEAS IN PRE-AP ALGEBRA 1

While the Pre-AP Algebra 1 framework is organized into four core units of study, the content is grounded in four big ideas, which are cross-cutting concepts that build conceptual understanding and spiral throughout the course. Since these ideas cut across units, they serve as the underlying foundation for the enduring understandings, key concepts, and learning objectives that make up the focus of each unit. A deep and productive understanding of the concepts presented in Pre-AP Algebra 1 relies on these four big ideas:

- **Patterns of Change:** Families of functions are uniquely defined by their patterns of change. Linear functions have a constant rate of change, quadratic functions have a linear rate of change, and exponential functions have a constant multiplicative rate of change.
- **Representations:** Functions and equations can be represented graphically, numerically (in tables), algebraically (with symbols), or verbally (in words). Algebraic forms of functions and equations can be purposefully manipulated into equivalent forms to reveal certain aspects of the function/equation.
- **Modeling with Functions:** Functions can be used to model real-world phenomena. A function derived from a real-world context can be manipulated free of its context, but the solution must be translated back in order to interpret its meaning in context.
- **Solutions:** A solution to an equation or inequality is a value or set of values that makes the equation or inequality true. Solutions can be found by applying rules of algebra to symbolic expressions, examining a graph of the equation or inequality, or testing numerical values.

OVERVIEW OF PRE-AP ALGEBRA 1 UNITS AND ENDURING UNDERSTANDINGS

Unit 1: Linear Functions and Equations	Unit 2: Exponential Functions and Expressions
▪ A linear function has a constant rate of change that can be visualized as the slope of the associated line. ▪ Linear functions can be written in different ways, and each one reveals different aspects of a function and the contextual scenario it represents. ▪ Linear equations and inequalities and systems of two linear equations or inequalities can model contextual scenarios with constraints. ▪ Solving a system of linear equations or inequalities is a process of determining the value or values that make the equations or inequalities in the system simultaneously true.	▪ An exponential function has constant multiplicative growth or decay. ▪ Exponential functions grow by equal factors over equal intervals. ▪ Exponential functions can model contextual scenarios that involve constant multiplicative growth or decay. ▪ Extending and applying the properties of exponents can help to make sense of scenarios involving exponential growth or decay. ▪ Graphs and tables can be used to estimate the solution to an equation that involves exponential expressions.
Unit 3: Quadratic Functions and Equations	**Unit 4: Comparing and Selecting Functions**
▪ A quadratic function has a rate of change given by a linear function. ▪ Quadratic functions can be expressed as a product of linear factors. ▪ Quadratic functions can be used to solve problems and model scenarios that involve a linear rate of change and symmetry around a unique minimum or maximum. ▪ Every quadratic equation, $ax^2 + bx + c = 0$ where a is not zero, has at most two real solutions. These solutions can be determined using the quadratic formula.	▪ Different function families have distinct properties and characteristics. ▪ Selecting an appropriate function to model a scenario often involves examining the context, data, assumptions, or constraints, and making comparisons to each function's distinct properties and characteristics. ▪ Functions and equations can be used to represent and solve problems related to a given mathematical or applied context. ▪ Functions can be used to model key characteristics of the relationship between variables in a data set.

Unit 1: Linear Functions and Equations

Suggested Timing: Approximately 10 weeks

Linear relationships are among the most prevalent and useful relationships in mathematics and the real world. While prior courses introduced students to foundational skills and knowledge about linear relationships and the concept of functions, this unit reinforces and deepens student understanding of constant rates of change, builds confidence in constructing linear functions, and emphasizes fluency in moving between algebraic forms of linear relationships. Students also learn function notation to describe the relationship between input and output values, relating inputs and outputs to domain and range.

Students relate arithmetic sequences to linear functions to extend their knowledge of linear relationships. They translate between different forms of equations and interpret key features of linear models. Students learn to use linear equations and linear inequalities in one and two variables to help reveal aspects of contextual scenarios and interpret the meaning of solutions in context.

The unit ends with students solving systems of linear equations and systems of linear inequalities in support of two goals: determining the solutions and becoming strategic and efficient in choosing appropriate methods for solving systems. Students use systems of linear equations and systems of linear inequalities to model physical phenomena, especially those with multiple constraints in which an optimal solution to an objective function is desired. Through a variety of applied contexts, students build upon their knowledge of solving systems of equations and develop more sophisticated understandings about what solutions to systems mean in contextual settings.

Through this unit, students deepen their skills associated with linear functions, equations, inequalities, and systems and appreciate the simplicity and power of linear relationships as building blocks of higher mathematics.

ENDURING UNDERSTANDINGS

Students will understand that …

- A linear function has a constant rate of change that can be visualized as the slope of the associated line.
- Linear functions can be written in different ways, and each one reveals different aspects of a function and the contextual scenario it represents.
- Linear equations and inequalities and systems of two linear equations or inequalities can model contextual scenarios with constraints.

- Solving a system of linear equations or inequalities is a process of determining the value or values that make the equations or inequalities in the system simultaneously true.

KEY CONCEPTS

- 1.1: Modeling with functions that have a constant rate of change
- 1.2: Representing linear functions in multiple ways
- 1.3: Modeling linear relationships with constraints
- 1.4: Solving systems of linear equations and linear inequalities

KEY CONCEPT 1.1: MODELING WITH FUNCTIONS THAT HAVE A CONSTANT RATE OF CHANGE

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.1.1 Determine whether a relationship is linear by identifying the constant rate of change.	**1.1.1a** If one variable changes by a constant difference as the other variable changes by one unit, the relationship is called linear with a constant rate of change. **1.1.1b** In a table of values that represents a linear relationship, when the input values differ by a constant amount, the corresponding output values will also differ by a constant amount. **1.1.1c** In a graph of a linear relationship, when the graph of the input-output pairs corresponds with a line, the slope of that line indicates its constant rate of change. **1.1.1d** The algebraic representation of a linear relationship between dependent variable y and independent variable x is given by $y = mx + b$ where m is the constant rate of change and b is the value of y when x is 0.
1.1.2 Construct and use arithmetic sequences to represent linear relationships.	**1.1.2a** Successive terms of an arithmetic sequence have a common difference equal to the constant rate of change. **1.1.2b** Starting from the first term in an arithmetic sequence with common difference d, each term can be found by adding d to the previous term. **1.1.2c** An arithmetic sequence is a linear relationship where the inputs are the whole numbers, and consecutive outputs, or terms, differ by a constant. **1.1.2d** At least one term, its position in the sequence, and the common difference between consecutive terms can be used to write an algebraic representation of an arithmetic sequence.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.1.3 Use algebraic notation to represent functions and sequences.	**1.1.3a** A function is a mathematical relationship where each input value is mapped to exactly one output value. The set of input values is called the domain of the function, and the set of output values is called the range of the function. **1.1.3b** Given a function named f, if x stands for any input value in the function's domain, then $f(x)$ stands for the output value that corresponds to that value of x. **1.1.3c** The graph of a function f is the graph of all ordered pairs (x, y), where $y = f(x)$. Each input-output pair for the function f satisfies this equation. **1.1.3d** The terms of a sequence can be denoted a_1, a_2, a_3, and so on. The nth term is represented by a_n where the subscript n indicates the position of that term in the sequence. **1.1.3e** A sequence is a function whose domain consists of the whole numbers. Each output of a function f that defines a sequence gives the term at position n whose value is $f(n)$.
1.1.4 Use function notation to determine inputs and outputs of functions.	**1.1.4a** The notation $f(a) = b$ implies that a is a valid input of function f, b is the corresponding output, and (a, b) is a point on the graph of f. **1.1.4b** An output value of a function f can be determined by evaluating the expression that defines $f(x)$ at a given input value. **1.1.4c** Input values of a function f may be determined by solving the equation $f(x) = d$ for x where d is a given output value.
1.1.5 Construct and apply linear functions to solve problems given mathematical or applied contexts.	**1.1.5a** A linear function f defined by $f(x) = mx + b$ has a constant rate of change m and an initial value b. **1.1.5b** Given two input-output pairs of a linear function $y = f(x)$, the constant rate of change of the function can be expressed as $\frac{\Delta y}{\Delta x}$ where Δx represents the change between two inputs, x_2 and x_1, and Δy represents the corresponding change between the two outputs, $f(x_2)$ and $f(x_1)$. **1.1.5c** In applied contexts, the inputs and outputs of linear functions must be interpreted in terms of the situation, which includes using the appropriate units.

Content Boundary: Students use arithmetic sequences to deepen their understanding of linear functions. Arithmetic sequences are linear functions in which the domain is the whole numbers. Memorizing specific formulas often associated with arithmetic sequences is *beyond the scope of this course*.

Cross Connection: Students come to Algebra 1 with prior knowledge about linear equations and linear functions. They are already familiar with $y = mx + b$ representing the graph of a line with slope m and y-intercept $(0, b)$. They also relate the equation $y = mx + b$ to a linear function with constant rate of change m and initial value b. This course deepens their understanding of rate of change and prepares them to compare linear, exponential, and quadratic functions. Students expand their understanding of functions to include a more sophisticated definition and terminology and to connect linear functions to arithmetic sequences. They also learn to use formal function notation that will be used in upcoming units and courses such as AP Precalculus, in which they will consider the idea of covariation in depth.

KEY CONCEPT 1.2: REPRESENTING LINEAR FUNCTIONS IN MULTIPLE WAYS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.2.1 Identify and interpret key characteristics of linear functions and their graphs.	**1.2.1a** The graph of $f(x) = mx + b$ lies on a line with slope m and y-intercept $(0, b)$. When a linear function's constant rate of change is 0, the graph lies on a horizontal line. **1.2.1b** If the graph of a linear function f has an x-intercept $(c, 0)$ and a y-intercept $(0, b)$, the solution to the equation $f(x) = 0$ is c, and $f(0) = b$. **1.2.1c** A function is increasing if, as the input values increase, the corresponding output values always increase. A function is decreasing if, as the input values increase, the corresponding output values always decrease. **1.2.1d** An increasing linear function has a positive rate of change, and a decreasing linear function has a negative rate of change.
1.2.2 Generate an algebraic representation of a linear function given a verbal description, a graphical representation, or a tabular representation.	**1.2.2a** An algebraic representation of a linear function can be expressed using two input-output pairs or the rate of change and one input-output pair. **1.2.2b** In a table of values that represents a linear function, the constant rate of change m is the difference in the output values when the corresponding input values increase by 1. Therefore, if two input values differ by k, then the two corresponding output values will differ by mk. **1.2.2c** Given a graph, the equation of a linear function can be determined using the line's slope and at least one other point on the line. **1.2.2d** A given representation of a linear function can be converted to another representation of the same linear function.
1.2.3 Translate between different algebraic forms of a linear equation.	**1.2.3a** The slope-intercept equation $y = mx + b$ represents a line with slope m and y-intercept $(0, b)$. **1.2.3b** When a line with slope m passes through the origin (0,0), its equation is $y = mx$. This equation represents a proportional relationship, also called direct variation. **1.2.3c** The point-slope form of a linear equation, which represents a line with slope m that passes through the point (x_1, y_1), is $y - y_1 = m(x - x_1)$ or $y = y_1 + m(x - x_1)$. **1.2.3d** The properties of algebra can be used to express equations for lines in different, but equivalent algebraic forms. In mathematical and applied contexts, different forms can reveal different characteristics and interpretations.

Cross Connection: Prior courses introduced students to the equations $y = mx + b$, which represents a line with slope m and y-intercept $(0, b)$, and $y = mx$, which represents a proportional relationship. As students expand their understanding of linear equations and functions to include different forms of a line and deepen their understanding of graphs of linear functions and sequences, they recognize additional applications of the relationships learned in prior grades.

KEY CONCEPT 1.3: MODELING LINEAR RELATIONSHIPS WITH CONSTRAINTS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.3.1 Construct and solve one-variable linear equations and inequalities that can be used to answer questions about a given mathematical or applied context.	**1.3.1a** A linear equation can have no solution, one solution, or infinitely many solutions. **1.3.1b** The solution to a linear inequality in one variable can be represented graphically on a number line or symbolically with an inequality. **1.3.1c** Solving a linear equation or inequality relies on a process of reasoning that starts from the assumption that the original equation (or inequality) has a solution, with each subsequent step maintaining equality with the previous step. **1.3.1d** Addition or subtraction by numbers and expressions and multiplication or division by a positive number applied to both sides of an inequality statement result in another inequality statement with the same solution set. **1.3.1e** Multiplying or dividing both sides of an inequality by −1 and changing the direction of the inequality symbol has the same effect as using the addition and subtraction properties to move all terms to the opposite side of the inequality symbol. **1.3.1f** In applied contexts, the solutions of linear equations and inequalities must be interpreted in terms of the situation, which includes using the appropriate units.
1.3.2 Construct and solve two-variable linear equations that model given mathematical or applied contexts.	**1.3.2a** The standard form of a two-variable linear equation is $Ax + By = C$, where A, B, and C are real numbers, and neither A nor B is 0. The solution set consists of all ordered pairs (x, y) that make the equation true. **1.3.2b** The graph of a two-variable linear equation lies on a line that represents all its solutions plotted in the coordinate plane. **1.3.2c** In applied contexts, a two-variable linear equation often represents a situation in which a constraint or condition exists and neither variable necessarily depends on the other. Solutions must be interpreted in terms of the situation, which includes using the appropriate units. **1.3.2d** Expressing a two-variable linear equation in terms of one of the variables can reveal information about the relationship between the variables.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.3.3 Construct and solve two-variable linear inequalities that model given mathematical or applied contexts.	**1.3.3a** A two-variable linear inequality relates two expressions such as $Ax + By$ and C with an inequality symbol <, >, ≤, or ≥, where A, B, and C are real numbers, and A and B are not both 0. The set of all ordered pairs (x, y) that make the inequality true is called the solution set. **1.3.3b** The solution set of a two-variable linear inequality is graphically represented by a half-plane. **1.3.3c** In applied contexts, a two-variable inequality often represents a situation involving resource limitations, goals, constraints, comparisons, and tolerances. Solutions must be interpreted in terms of the situation, which includes using the appropriate units. **1.3.3d** Expressing a two-variable linear inequality in terms of one of the variables can reveal information about how the quantities in the linear inequality relate to each other.

Content Boundary: Applications of two-variable linear inequalities will be addressed further in Unit 4. This key concept emphasizes determining whether a given ordered pair is a solution to a linear inequality and graphing the solution set on a coordinate plane.

Cross Connection: In this key concept, students regard the two variables in a linear equation as two independent quantities related by a constraint; this is distinct from the input–output thinking that characterizes the functional relationships between the quantities in the previous key concept. Students connect the one-variable equations they solved in middle school to the two-variable relationships they encounter in this unit. They also continue to deepen their understanding of equation solving as a process of reasoning by understanding that both one- and two-variable equations are statements that can be either true or false.

KEY CONCEPT 1.4: SOLVING SYSTEMS OF LINEAR EQUATIONS AND LINEAR INEQUALITIES

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.4.1 Solve systems of linear equations algebraically or by estimating solutions either graphically or numerically.	**1.4.1a** A solution to a system of two linear equations is an ordered pair that satisfies both equations simultaneously. **1.4.1b** The solution(s) to a system of two linear equations corresponds to the point(s) of intersection of their graphs. **1.4.1c** Algebraically equivalent expressions can be substituted for one another in an equation. **1.4.1d** Given a system of two linear equations, replacing one equation with the sum of that equation and a multiple of the other produces an equivalent system that has the same solution set. **1.4.1e** The structures of the two linear equations in a system often suggest whether the substitution or elimination method is a more efficient way to solve the system.
1.4.2 Determine the number of solutions to a system of two linear equations.	**1.4.2a** A system of two linear equations has one solution when the corresponding lines intersect at one point because they have different slopes. **1.4.2b** A system of two linear equations has infinitely many solutions when the corresponding lines coincide. The two linear equations will have the same slope and the same y-intercepts. **1.4.2c** A system of two linear equations has no solution when the corresponding lines are parallel. The two linear equations will have the same slope and different y-intercepts.
1.4.3 Construct and solve systems of two linear equations that model given mathematical or applied contexts.	**1.4.3a** A system of two linear equations can be used to model situations in which two quantities are subject to two or more constraints. **1.4.3b** A system of two linear equations can be used to determine when two linear functions have the same input-output pair(s), if any. **1.4.3c** In applied contexts, the solution to a system of two linear equations can be determined mathematically but must be interpreted in terms of the situation, which includes using the appropriate units.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.4.4 Construct and solve systems of linear inequalities that model mathematical or applied contexts.	**1.4.4a** A solution to a system of linear inequalities is an ordered pair that satisfies all the inequalities in the system simultaneously. **1.4.4b** The solution to a system of linear inequalities can be represented graphically as the intersection of half-planes that represent the solutions to the individual inequalities in the system. Every ordered pair that satisfies all the inequalities in the system is contained in this shared region. **1.4.4c** A system of linear inequalities can be used to model situations in which two quantities are subject to multiple constraints. **1.4.4d** In applied contexts, the solution to a system of linear inequalities can be determined mathematically but must be interpreted in terms of the situation, which includes using the appropriate units.

Content Boundary: In this unit, students work with systems of linear equations and inequalities in two variables. Systems with three or more variables are *beyond the scope of this course*.

Content Connection: Students build upon their knowledge of systems of equations from Grade 8 by considering in more detail the logical reasoning that allows for algebraic methods such as substitution, elimination, or addition to produce a system with the same solution set as the original system. Students connect the intersection of the graphs of two functions, f and g, to the solutions to a system of equations $y = f(x)$ and $y = g(x)$ and the equation $f(x) = g(x)$.

Cross Connection: The focus of this key concept is to become strategic and efficient about choosing a particular method to solve a system of equations. Students understand that graphical and tabular methods of solving a system of equations can be imprecise and appreciate that the elimination method is convenient for linear equations written in standard form. However, elimination is not generally applicable for nonlinear systems. The substitution method has wider utility for nonlinear systems and will be explored in Unit 4. Other algebraic techniques, such as those involving matrices, are *beyond the scope of this course*.

Cross Connection: Students make connections from their prior knowledge to understand that scenarios involving inequalities are often best represented graphically. That is, the solution to a one-variable linear inequality can be displayed as a shaded portion on the number line, the solution to a two-variable linear inequality can be displayed as a half-plane, and the solution to a system of two-variable linear inequalities can be displayed as the intersection of the associated half-planes. Any of these graphical representations allows specific values of the solution set to be quickly identified.

Unit 2: Exponential Functions and Expressions

Suggested Timing: Approximately 6 weeks

Students explore the pattern of change in exponential functions by examining graphs, tables, and sequences. They first consider sequences that grow through multiplicative patterns rather than additive patterns. Once students recognize that the pattern of change for an exponential function is multiplication by a common factor, they progress to constructing representations of exponential growth and decay functions.

The rules of exponents, and their extension to rational exponents, support students to make sense of the relationship between exponential decay and negative exponents, as well as to understand why it makes sense that the domain of exponential functions is all real numbers and why their graphs are continuous.

The unit culminates with students further investigating how exponential functions can model physical phenomena that exhibit constant multiplicative change, specifically over intervals in which the difference in the inputs is greater than or less than one. Exponential functions are presented as multiplicative analogues of linear functions. Thus, an enduring connection can be drawn between these two classes of functions and their shared properties.

ENDURING UNDERSTANDINGS

Students will understand that ...

- An exponential function has constant multiplicative growth or decay.
- Exponential functions grow by equal factors over equal intervals.
- Exponential functions can model contextual scenarios that involve constant multiplicative growth or decay.
- Extending and applying the properties of exponents can help to make sense of scenarios involving exponential growth or decay.
- Graphs and tables can be used to estimate the solution to an equation that involves exponential expressions.

KEY CONCEPTS

- 2.1: Modeling with functions that have multiplicative growth or decay
- 2.2: Interpreting equivalent exponential expressions
- 2.3: Extending multiplicative patterns of change

KEY CONCEPT 2.1: MODELING WITH FUNCTIONS THAT HAVE MULTIPLICATIVE GROWTH OR DECAY

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
2.1.1 Determine whether a relationship is exponential by identifying the constant proportional change.	**2.1.1a** If one variable changes by a constant factor as the other variable changes by one unit, the relationship is called exponential. **2.1.1b** In a table of values that represents an exponential relationship, when the input values differ by a constant amount, the corresponding output values differ by a constant factor.
2.1.2 Construct and use geometric sequences to represent exponential relationships.	**2.1.2a** Successive terms of a geometric sequence have a common ratio equal to the constant factor. **2.1.2b** Starting from the first term in a geometric sequence with common ratio r, each term can be found by multiplying the previous term by r. **2.1.2c** A geometric sequence is defined by an exponential relationship where the inputs are the whole numbers and the ratio of consecutive outputs, or terms, is constant. **2.1.2d** At least one term, its position in the sequence, and the common ratio between consecutive terms can be used to write an algebraic representation of a geometric sequence.
2.1.3 Construct and apply exponential functions to solve problems given mathematical or applied contexts.	**2.1.3a** An exponential function f defined by $f(x) = a(b)^x$ has a constant factor b and an initial value a, where b is positive and $b \neq 1$. When $0 < b < 1$, b is called the decay factor, and when $b > 1$, b is called the growth factor. **2.1.3b** Given an exponential function f, $\frac{f(n+1)}{f(n)}$ gives the growth (or decay) factor. **2.1.3c** In applied contexts, the inputs and outputs must be interpreted in terms of the situation, which includes using the appropriate units.

Content Boundary: In this key concept, students use their knowledge of multiplicative patterns to explore simple exponential growth and decay relationships. The goal of introducing students to geometric sequences is to investigate multiplicative patterns as a counterpoint to the additive patterns of linear functions and arithmetic sequences. Students are expected to generate and connect different representations of exponential functions and geometric sequences.

Content Boundary: In Key Concept 2.1, representations of exponential functions should have inputs that differ by one, but these inputs don't need to be integers. Inputs differing by more than one are covered in Key Concept 2.3.

Content Boundary: Knowing formulas associated with geometric sequences and using logarithms to find exact solutions to problems is *beyond the scope of this course*.

Content Boundary: In the course, students will only explore exponential functions of the form $f(x) = a(b)^x$ for $a > 0$. Students will explore functions where $a < 0$ in Algebra 2 in the context of transformation of functions.

KEY CONCEPT 2.2: INTERPRETING EQUIVALENT EXPONENTIAL EXPRESSIONS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.2.1 Use exponent rules to rewrite expressions with rational exponents in equivalent forms.	**2.2.1a** Exponential expressions can be rewritten in equivalent forms using the multiplication, division, and power of powers rules. **2.2.1b** Application of the rules of exponents lead to $n^0 = 1$ and $n^{-k} = \frac{1}{n^k}$ for $n \neq 0$. **2.2.1c** The properties of rational exponents are extensions of the properties of integer exponents. **2.2.1d** The kth root of n can be expressed using the exponent $\frac{1}{k}$. That is, $\sqrt[k]{n} = n^{\frac{1}{k}}$ for integers n and k, where $n \geq 0$ and $k > 0$. **2.2.1e** Application of the rules of exponents leads to $n^{\frac{a}{b}} = \left(n^{\frac{1}{b}}\right)^a = \left(\sqrt[b]{n}\right)^a$ for a nonnegative number n and integers a and b such that $b \neq 0$.
2.2.2 Rewrite exponential expressions representing a mathematical or applied context to solve problems.	**2.2.2a** The properties of exponents can be used to change the base of an exponential expression. **2.2.2b** If b is not −1, 0, or 1, and $b^u = b^v$, then $u = v$. Thus, when two exponential expressions are equal and have the same base, the exponents must be equal. **2.2.2c** In applied contexts, exponent rules can be applied to understand and interpret exponential growth over different input intervals.

Content Boundary: Students are not expected to simplify complicated quotients, such as $\frac{36x^3y^{-4}z^7}{72x^{-5}y^6z^8}$, because these expressions have limited usefulness in the contextual scenarios they will encounter.

Content Boundary: The translation between algebraic forms of exponential functions should be limited to those that can be performed using the exponent rules described in Key Concept 2.2. Algebraic manipulations that require logarithms should not be included and go *beyond the scope of this course*.

Content Boundary: In this course, students are only expected to estimate solutions to exponential equations in one variable. In Algebra 2, students will use logarithms to determine the exact solutions to exponential equations in one variable.

Cross Connection: Students learned about negative exponents and scientific notation in prior courses. By revisiting and extending these properties to include rational exponents, students have an opportunity to deepen both their conceptual understanding and their procedural fluency. Students should see the rules of exponents as extensions of the properties of multiplication and division.

Cross Connection: Students work toward flexibly and fluently translating among different forms of expressions involving exponents. For example, often in AP Calculus it is advantageous to express the function $f(x) = \frac{1}{x^2}$ as $f(x) = x^{-2}$ or similarly, to express the function $g(x) = \sqrt{x}$ as $g(x) = x^{\frac{1}{2}}$ to find their derivatives or integrals.

KEY CONCEPT 2.3: EXTENDING MULTIPLICATIVE PATTERNS OF CHANGE

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
2.3.1 Describe key characteristics of exponential functions and their graphs.	**2.3.1a** The graph of $f(x) = a(b)^x$ is a curve with a y-intercept of $(0, a)$ and for which $f(x) > 0$. **2.3.1b** An increasing exponential function has a growth factor, and a decreasing exponential function has a decay factor. **2.3.1c** The growth (or decay) factor of an exponential function can be expressed as a percent change. So, a growth factor of p means $f(n+1) = p \cdot f(n)$ or $f(n+1)$ is $100 \cdot p\%$ of $f(n)$. **2.3.1d** In an exponential function: ▪ If the decay factor is b, then $1 - b$ represents the percent decrease. ▪ If the growth factor is b, then $b - 1$ represents the percent increase. **2.3.1e** The intersection point, if any, of the graph of an exponential function, f, and a horizontal line $y = d$ can be used to solve the equation $f(x) = d$ because the x-coordinate of the intersection point satisfies the equation.
2.3.2 Describe how the output of an exponential function behaves when the input changes by more or less than one.	**2.3.2a** When an exponential function's inputs differ by n, the value of $\frac{f(a+n)}{f(a)}$ is the growth (or decay) factor raised to the power of n. **2.3.2b** Given an exponential function, if the output values change by a factor of b when the input values increase by 1, then the output values will change by a factor of b^n when the input values increase by n.
2.3.3 Generate an algebraic representation of an exponential function given a verbal description, a graphical representation, or a tabular representation.	**2.3.3a** For an exponential function, if any two output values change by a factor of c when the corresponding input values increase by n, then the growth (or decay) factor is $c^{\frac{1}{n}}$, where n is a positive rational number. **2.3.3b** The algebraic representation of an exponential function can be determined by: ▪ The growth (or decay) factor b, and any point on the graph of the function. ▪ Any 2 points on the graph of the function. ▪ Any 2 coordinate pairs in a table or given numerically that satisfy the function. **2.3.3c** A given representation of an exponential function can be converted to another representation of the same exponential function.

Content Boundary: Students express, interpret, and translate between exponential growth or decay whether on intervals that increase by 1-unit or *n*-units. When considering percent growth or decay, students must first go from a given percent increase/decrease to a growth/decay factor before considering the change of different input intervals.

Unit 3: Quadratic Functions and Equations

Suggested Timing: Approximately 9 weeks

In this unit, students develop a strong foundation in the concept of quadratic functions. Students learn that quadratic functions have a linear rate of change and are often formed by multiplying two linear expressions, demonstrating that they are not themselves linear. Quadratic functions are useful for modeling phenomena that have a linear rate of change and exhibit symmetry across a vertical axis through a unique minimum or maximum. This foundational understanding of quadratic functions and equations helps students build their conceptual knowledge of nonlinear functions and prepares them for further study of polynomial and rational functions in Algebra 2.

ENDURING UNDERSTANDINGS

Students will understand that …

- A quadratic function has a rate of change given by a linear function.
- Quadratic functions can be expressed as a product of linear factors.
- Quadratic functions can be used to solve problems and model scenarios that involve a linear rate of change and symmetry around a unique minimum or maximum.
- Every quadratic equation, $ax^2 + bx + c = 0$ where a is not zero, has at most two real solutions. These solutions can be determined using the quadratic formula.

KEY CONCEPTS

- 3.1: Modeling with functions that have a linear rate of change
- 3.2: Representing quadratic functions and equations in multiple ways
- 3.3: Solving quadratic equations

KEY CONCEPT 3.1: MODELING WITH FUNCTIONS THAT HAVE A LINEAR RATE OF CHANGE

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.1.1 Determine whether a relationship is quadratic by identifying its linear rate of change.	**3.1.1a** In a quadratic relationship, the differences in the outputs over equal intervals have a linear rate of change. In other words, the second differences are constant. **3.1.1b** The patterns of change in a quadratic relationship can be observed in a table, graph, or sequence most easily by examining the change in outputs as the inputs increase by one.
3.1.2 Construct and apply quadratic functions to solve problems given a mathematical or applied context.	**3.1.2a** A quadratic sequence is a quadratic function whose domain is the whole numbers. Successive terms in a quadratic sequence can be obtained using the first or second differences of the corresponding quadratic function. **3.1.2b** All quadratic functions can be expressed in a form equivalent to $f(x) = ax^2 + bx + c$ where $a \neq 0$. That is, all quadratic functions contain a squared term. **3.1.2c** In applied contexts, a quadratic function often represents projectile motion, other physical phenomena modeled graphically by a U-shaped curve, or situations involving the product of two linear factors. **3.1.2d** In applied contexts, the inputs and outputs of quadratic functions must be interpreted in terms of the situation, which includes using the appropriate units.

Cross Connection: This key concept motivates students to see the need for a new type of nonlinear function to represent scenarios that cannot be adequately modeled by a linear or exponential function, such as the effect of gravity or measures of area with dimensions modeled by linear expressions. Students learn that any scenario, physical or otherwise, that involves multiplying two linear expressions will be quadratic. Earlier in Algebra 1, students saw that physical scenarios that exhibit a constant rate of change can be modeled by a linear function. In AP Precalculus, AP Calculus, and AP Physics, students will use quadratic functions to model particle motion problems that exhibit constant acceleration.

KEY CONCEPT 3.2: REPRESENTING QUADRATIC FUNCTIONS AND EQUATIONS IN MULTIPLE WAYS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.2.1 Interpret features of a quadratic function in terms of a given mathematical or applied context.	**3.2.1a** The graph of a quadratic function lies on a curve called a parabola with an equation that can be expressed in the form $y = ax^2 + bx + c$, where $a \neq 0$. **3.2.1b** Given a quadratic function *f*: ▪ The real zeros of *f* correspond to the *x*-intercepts of its graph. ▪ The maximum or minimum value of *f* corresponds to the *y*-coordinate of the vertex of its graph. ▪ The function *f* changes from increasing to decreasing or from decreasing to increasing at the *x*-value of the vertex of its graph. **3.2.1c** In applied contexts, the features of a quadratic function and its graph depend on the contextual domain. **3.2.1d** Key features of the graph of a quadratic function, such as intercepts and maximum or minimum values, often represent constraints in an applied context.
3.2.2 Describe the key features of the graph of a quadratic equation.	**3.2.2a** The graph of a quadratic equation, called a parabola, has an axis of symmetry, a vertex, a *y*-intercept, and either 0, 1, or 2 *x*-intercepts. **3.2.2b** Given the graph of a quadratic equation, $y = ax^2 + bx + c$, where $a \neq 0$: ▪ The graph intercepts the *y*-axis at *c*. ▪ The graph intercepts the *x*-axis at the real solutions to $ax^2 + bx + c = 0$. **3.2.2c** When the graph of a quadratic function has two *x*-intercepts, the *x*-coordinate of the vertex is located halfway between them.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.2.3 Use different forms of a quadratic function to reveal features of its graph.	**3.2.3a** Common forms of the equation of a quadratic function include standard form, factored form, and vertex form, and each form reveals different features of the function and its graph. **3.2.3b** For a quadratic equation in factored form, $y = a(x - r)(x - s)$, the coordinates of the x-intercepts are $(r, 0)$ and $(s, 0)$. When $r = s$, there is only one x-intercept. **3.2.3c** For a quadratic equation in vertex form, $y = a(x - h)^2 + k$, the coordinates of the vertex are (h, k). **3.2.3d** For a quadratic equation in standard form, $y = ax^2 + bx + c$, the x-coordinate of the vertex is $-\frac{b}{2a}$, and the y-intercept is $(0, c)$. **3.2.3e** For all forms of a quadratic equation: ▪ When $a > 0$, the graph is concave up. ▪ When $a < 0$, the graph is concave down.
3.2.4 Generate an algebraic representation of a quadratic function given a verbal description, a graphical representation, a tabular representation, or certain key points.	**3.2.4a** The algebraic representation of a quadratic function can be constructed using its vertex and one other input-output pair. **3.2.4b** The algebraic representation of a quadratic function can be constructed from two x-intercepts of its graph and one other input-output pair. **3.2.4c** Any three points, not all on the same line, can be used to determine a unique quadratic function whose graph includes those points. **3.2.4d** A given representation of a quadratic function can be converted to another representation of the same quadratic function.

Content Boundary: This course only addresses parabolas that open upward or downward because the equation of a parabola is only a function of x when oriented in this way. Parabolas oriented in other directions and their associated equations are *beyond the scope of this course*. The geometric definition of a parabola in terms of its focus and directrix is also *beyond the scope of this course*.

Cross Connection: A quadratic equation can be expressed in many equivalent forms, and each form reveals or obscures certain features of the quadratic function it defines. This understanding builds on the work students did with linear equations in Unit 1. Students translate between these forms and choose the representation that best serves the problem at hand. Procedural fluency in selecting and rewriting expressions for different purposes is a skill needed for Algebra 2 and AP Precalculus as students investigate the structure of polynomial and rational functions. By observing the usefulness of different forms, students recognize this work as useful rather than repeated procedural practice.

KEY CONCEPT 3.3: SOLVING QUADRATIC EQUATIONS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.3.1 Solve a quadratic equation graphically and interpret the solution in terms of the graph of the associated quadratic function.	**3.3.1a** Given a quadratic function f, the solutions, if any, to the equation $f(x) = d$ are the x-coordinates of the points where the graphs of $y = f(x)$ and $y = d$ intersect. **3.3.1b** Given a quadratic function f, the solutions to the equation $f(x) = 0$ are the x-coordinates where the graph of $y = f(x)$ intercepts the x-axis.
3.3.2 Rewrite a quadratic expression in different, but equivalent, forms.	**3.3.2a** Rewriting quadratic expressions in different but equivalent forms supports solving equations and analyzing functions and their graphs. **3.3.2b** Quadratic expressions can be rewritten in different forms by applying the distributive property. **3.3.2c** A quadratic expression in standard form can be rewritten in vertex form by completing the square. **3.3.2d** For a quadratic function f, the standard form expression, $ax^2 + bx + c$, and the vertex form expression, $a(x - h)^2 + k$, are equivalent when $h = -\frac{b}{2a}$ and $k = f\left(-\frac{b}{2a}\right)$. So, the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
3.3.3 Solve quadratic equations using algebraic techniques.	**3.3.3a** For any positive real number a, the equation $x^2 = a$ has two solutions: $\sqrt{a}$ and $-\sqrt{a}$. **3.3.3b** There is no real number that satisfies the equation $x^2 = a$ when a is negative, and there is one real number, 0, that satisfies the equation $x^2 = 0$. **3.3.3c** Any quadratic equation can be expressed in the form $a(x - h)^2 + k = 0$ by completing the square. **3.3.3d** The zero-product property applies to quadratic equations. If $a(x - r)(x - s) = 0$ for nonzero values of a and $r \neq s$, then either $x - r = 0$ or $x - s = 0$. The solutions to the original equation are r and s. **3.3.3e** Expressing $ax^2 + bx + c$ as the product of two linear factors in the equation $ax^2 + bx + c = 0$ leads to equations that can be solved using the zero-product property.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.3.4 Solve quadratic equations using the quadratic formula.	**3.3.4a** The quadratic formula is derived by solving the equation $ax^2 + bx + c = 0$, where $a \neq 0$, by completing the square. **3.3.4b** For the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, the real solutions, as given by the quadratic formula, are $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.
3.3.5 Determine the number of real solutions to a quadratic equation.	**3.3.5a** Every quadratic equation has 0, 1, or 2 real solutions. **3.3.5b** The value of the discriminant, $b^2 - 4ac$, can be used to determine the number of real solutions to the quadratic equation $ax^2 + bx + c = 0$: ▪ If $b^2 - 4ac = 0$, there is 1 real solution. ▪ If $b^2 - 4ac > 0$, there are 2 real solutions. ▪ If $b^2 - 4ac < 0$, there are no real solutions. **3.3.5c** The number of distinct real solutions to the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$ is the same as the number of x-intercepts of the graph of $y = ax^2 + bx + c$. **3.3.5d** The solutions to a quadratic equation $ax^2 + bx + c = 0$, when written in the form $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$, reveal: ▪ The axis of symmetry has the equation $x = -\frac{b}{2a}$. ▪ The x-intercept(s) are a horizontal distance of $\pm \frac{\sqrt{b^2 - 4ac}}{2a}$ from the axis of symmetry.
3.3.6 Construct and apply quadratic equations to solve problems given a mathematical or applied context.	**3.3.6a** In applied contexts, a quadratic equation is used to solve problems involving projectile motion, other physical phenomena modeled by a parabola, or situations involving the product of two linear factors. **3.3.6b** In applied contexts, the solutions to a quadratic equation can be determined algebraically but must be interpreted in terms of the situation, which includes using the appropriate units.

Content Boundary: This key concept focuses on students making explicit connections between the real number solutions of a quadratic equation and the x-intercept(s) of the associated graph. In Algebra 2, students will learn that all quadratic equations can be solved, but some quadratic equations require a new number system to adequately express the solution set. Specifically, the complex number system and complex solutions to quadratic equations are *beyond the scope of this course*.

Unit 4: Comparing and Selecting Functions

Suggested Timing: Approximately 4 weeks

In this unit, students compare the characteristics of the three types of functions they have explored so far in the course: linear, exponential, and quadratic functions. Students begin by extending their understanding of the arithmetic, geometric, and quadratic sequences related to these function types by considering mathematical and contextual applications in which recursive definitions are desired. They then identify the patterns of change in contextual scenarios to determine whether linear, exponential, or quadratic functions provide the most appropriate models. Students also apply their knowledge of functions to examine scenarios modeled by piecewise-defined functions, although writing the equations for such function models is saved for Algebra 2.

Next, students develop and solve systems of equations and inequalities involving one or more types of functions to model phenomena, including those with multiple constraints in which an optimal solution to an objective function is desired. They also connect the intersection points of graphs of multiple functions to the solutions of their related systems. Through these contexts, students build upon their prior knowledge of solving systems and develop a more sophisticated understanding of what the solution or solutions to a system of equations or inequalities mean in the context of the problem.

Students then consider whether the trends in a given data set are best modeled by a linear, quadratic, or exponential function. Finally, they construct and apply function models to analyze data sets, and they interpret their findings in context.

ENDURING UNDERSTANDINGS

Students will understand that ...

- Different function families have distinct properties and characteristics.
- Selecting an appropriate function to model a scenario often involves examining the context, data, assumptions, or constraints, and making comparisons to each function's distinct properties and characteristics.
- Functions and equations can be used to represent and solve problems related to a given mathematical or applied context.
- Functions can be used to model key characteristics of the relationship between variables in a data set.

KEY CONCEPTS

- 4.1: Comparing and analyzing functions
- 4.2: Using functions to model contextual scenarios
- 4.3: Using functions to model data sets

KEY CONCEPT 4.1: COMPARING AND ANALYZING FUNCTIONS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.1.1 Construct and use recursive forms of sequences in mathematical and applied contexts.	**4.1.1a** Defining a sequence recursively requires at least one term, the starting index value, and a rule for generating the next term. **4.1.1b** The recursive formulas for arithmetic, geometric, and quadratic sequences reflect specific patterns of change: ▪ To generate a term in an arithmetic sequence, add the common difference to the previous term. ▪ To generate a term in a geometric sequence, multiply the previous term by the common ratio. ▪ To generate a term in a quadratic sequence, add the next term of the arithmetic sequence that defines the differences to the previous term.
4.1.2 Compare properties of exponential, linear, and quadratic functions.	**4.1.2a** Linear growth is characterized by addition of the same constant to each value of the domain. **4.1.2b** Exponential growth is characterized by multiplication by the same factor to each value of the domain. **4.1.2c** Linear functions grow by equal differences over equal intervals, and exponential functions grow by equal factors over equal intervals. **4.1.2d** Beyond the minimum value, quadratic growth is characterized by addition of a linear sequence to each value of the domain. **4.1.2e** For large enough input values, the outputs of an exponential growth function will eventually exceed those of a linear or quadratic function.
4.1.3 Compare properties of two or more functions represented in different ways.	**4.1.3a** Each x-coordinate of an intersection point of the graphs of functions f and g is a solution to the equation $f(x) = g(x)$. **4.1.3b** Given two functions f and g: ▪ For x-intervals where $f(x) > g(x)$, the graph of f appears above the graph of g. ▪ For x-intervals where $f(x) < g(x)$, the graph of f appears below the graph of g. **4.1.3c** Two or more functions can be compared even when each one is represented in a different way. Observing key features such as zeros, intercepts, intervals of increase/decrease, patterns of change, or maximum and minimum points across different representations allows comparisons of two or more functions.

Content Boundary: The study of end behavior in general and specifically the use of limit notation is *beyond the scope of this course*.

Cross Connection: Now that students are familiar with the three types of functions that form the foundation of Algebra 1, they can begin to compare the behaviors of these function types, especially in scenarios involving more than one type of function. This work will be expanded upon in AP Precalculus and AP Calculus, where they will regularly encounter functions built from components of other functions, such as $f(x) = e^2 + x^2$ or $g(x) = \frac{x+1}{x^2+2x+2}$. By examining the behavior of linear, exponential, and quadratic functions across the real number domain, students begin to consider the end behavior of functions, preparing them for their work in Algebra 2 and AP Precalculus.

Cross Connection: The inclusion of recursive forms of sequences provides a connection to the algorithmic thinking skills students use in AP Computer Science Principles. Representing and using sequences in this form builds on students' work in earlier units where they studied the closed forms of sequences.

KEY CONCEPT 4.2: USING FUNCTIONS TO MODEL CONTEXTUAL SCENARIOS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.2.1 Construct a linear, exponential, or quadratic function to appropriately represent an applied context.	**4.2.1a** Identifying patterns of change can help to select and construct an appropriate function to model a given situation. ▪ Scenarios involving additive growth are best represented by linear functions. ▪ Scenarios involving multiplicative growth are best represented by exponential functions. ▪ Scenarios involving additive growth that changes at a constant rate are best represented by quadratic functions. **4.2.1b** Representing a situation numerically, graphically, or by drawing a diagram can help to select an appropriate function to model a given situation.
4.2.2 Use a piecewise-defined function that represents an applied context to solve problems.	**4.2.2a** A piecewise-defined function consists of two or more functions, each restricted to non-overlapping intervals of the domain. **4.2.2b** A contextual scenario that has different characteristics over different intervals, such as different rates of change, can be modeled using a piecewise-defined function. **4.2.2c** The output value of a piecewise-defined function can be estimated, or sometimes determined, from a graph of the function.
4.2.3 Solve problems involving piecewise-defined, linear, exponential, or quadratic functions that represent an applied context.	**4.2.3a** Scenarios involving linear and/or quadratic equations can be solved algebraically or with the aid of technology. **4.2.3b** Scenarios involving exponential equations or piecewise-defined functions can be represented and solved graphically, often with the aid of technology. **4.2.3c** The domain of a function must make sense within an applied context. **4.2.3d** Solutions to problems involving quantities related by functions must be interpreted in terms of the situation, which includes using the appropriate units.
4.2.4 Construct one or more equations or inequalities to represent an applied context.	**4.2.4a** One or more equations or inequalities can be used to represent relationships between quantities and constraints in a situation. **4.2.4b** Representing a given situation in multiple ways can lead to the equations or inequalities needed to solve a problem.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
4.2.5 Solve problems involving two or more equations or inequalities that represent an applied context.	**4.2.5a** Systems of equations can be solved algebraically or graphically depending on the types of equations that make up the system. **4.2.5b** Systems of inequalities and their solution sets can be represented graphically, and individual solutions can be identified using the graphs. **4.2.5c** Solutions to problems involving quantities related by systems of equations or inequalities must be interpreted in terms of the situation, which includes using the appropriate units.

Content Boundary: Students are not expected to determine an algebraic representation for a piecewise-defined function given a different representation. They should be able to use and analyze given piecewise-defined functions and be able to graph a simple piecewise defined function with linear components.

Cross Connection: Students deepen their understanding of the different function families studied in the previous units and apply their knowledge to decide which type of function best represents a scenario. Working with systems that are not limited to linear equations or inequalities extends students' work from Unit 1. Piecewise-defined functions are introduced in this unit and will be important in later courses where students use them to model contextual scenarios or analyze function behavior, and to represent concepts such as continuity and limits.

KEY CONCEPT 4.3: USING FUNCTIONS TO MODEL DATA SETS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.3.1 Select an appropriate function model given a data set.	**4.3.1a** A scatterplot can be analyzed to determine whether a relationship exists between the numerical quantities in a bivariate data set. **4.3.1b** Linear functions can be used to model data sets that exhibit a roughly constant rate of change. **4.3.1c** Exponential functions can be used to model data sets that exhibit a roughly constant growth or decay factor. **4.3.1d** Quadratic functions can be used to model data sets that exhibit rates of change roughly modeled by a linear function. Such data sets are roughly symmetrical and feature either a maximum or minimum value.
4.3.2 Construct an appropriate function model given a data set.	**4.3.2a** A function that models the quantities in a bivariate data set can be derived using two or three representative points. **4.3.2b** An appropriate function model matches the trends in the data set. The closer that most points of the scatterplot are to the graph of a function, the better that model fits the data set.
4.3.3 Solve problems involving linear, exponential, or quadratic functions that model a given data set.	**4.3.3a** Features of a function model can be interpreted in terms of the quantities represented by the data set. **4.3.3b** Since functions may not model a data set perfectly, values predicted using the function may differ from the actual data values. **4.3.3c** A function model can be used to predict an output for a given input. **4.3.3d** A function model based on a data set can be a poor model for predicting output values when using input values outside the domain of the data set.

Content Boundary: In Algebra 1, students are only expected to use the key features of the graphs of linear, quadratic, and exponential functions to determine a model to represent the trends in a scatterplot. Students are expected to use graphing technology to explore which function appropriately models the trends in the scatterplot. Using statistical tools embedded in technology (e.g., regression) to determine a function model is *beyond the scope of the course*.

Content Boundary: Each data set used in this course is carefully chosen to make the appropriate function model relatively straightforward for students to recognize based on the trends in the corresponding scatterplot.

Cross Connection: This key concept sets the foundation for students to use linear, quadratic, and exponential functions to model trends in scatterplots. In Algebra 2, students extend these understandings by using technology and statistical tools like regression to determine an appropriate function model for a given data set.

Pre-AP Algebra 1 Model Lessons

Model lessons in Pre-AP Algebra 1 are developed in collaboration with Algebra 1 educators across the country and are rooted in the course framework, shared principles, and areas of focus. Model lessons are carefully designed to illustrate on-grade-level instruction. Pre-AP strongly encourages teachers to internalize the lessons and then offer the supports, extensions, and adaptations necessary to help all students reach these on-grade-level goals.

The purpose of these model lessons is twofold:

- **Robust instructional support for teachers:** Pre-AP Algebra 1 model lessons are comprehensive lesson plans that, along with accompanying student resources, embody the Pre-AP approach to teaching and learning. Model lessons provide clear and substantial instructional guidance to support teachers as they engage students in the shared principles and areas of focus.
- **Key instructional strategies:** Commentary and analysis embedded in each lesson highlight not just what students and teachers do in the lesson, but also how and why they do it. This educative approach provides a way for teachers to gain unique insight into key instructional moves that are powerfully aligned with the Pre-AP approach to teaching and learning. In this way, each model lesson works to support teachers in the moment of use with students in their classroom.

Teachers have the option to use any or all model lessons alongside their own locally developed instructional resources. Model lessons target content areas that tend to be challenging for teachers and students. While the lessons are distributed throughout all four units, they are concentrated more heavily in the beginning of the course to support teachers and students in establishing a strong foundation in the Pre-AP approach to teaching and learning.

SUPPORT FEATURES IN MODEL LESSONS

The following support features recur throughout the Pre-AP Algebra 1 lessons to promote teacher understanding of the lesson design and provide direct-to-teacher strategies for adapting lessons to meet their students' needs:

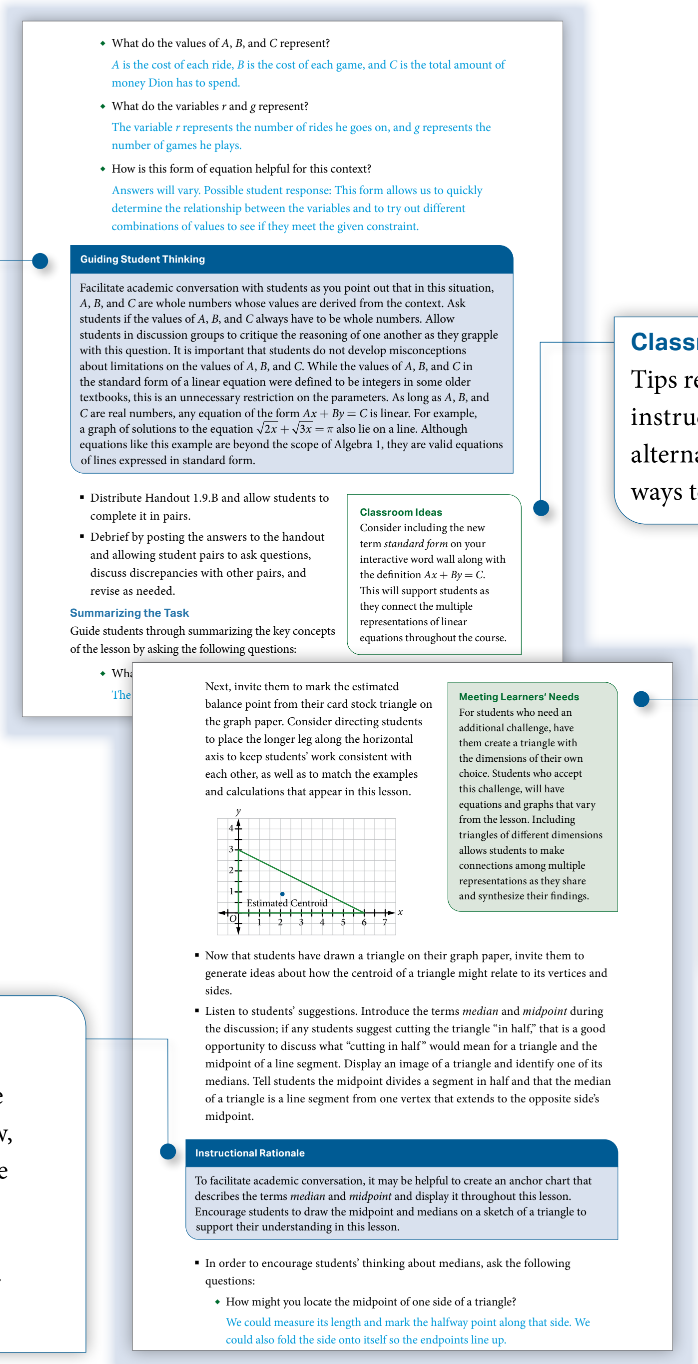
- Instructional Rationale
- Guiding Student Thinking
- Meeting Learners' Needs
- Classroom Ideas

Guiding Student Thinking: Ways to facilitate productive student thinking and prevent or address student misconceptions in critical areas of the lesson.

Classroom Ideas: Tips related to the logistics of the instruction, such as suggestions for alternative presentation methods or ways to alleviate pacing concerns.

Meeting Learners' Needs: Optional strategies to address needs of multilingual learners, students with disabilities, or others who would benefit from enhancements and adjustments to improve access and readiness.

Instructional Rationale: Insight into the strategic design and purpose of the instructional choices, flow, and scaffolding within the model lesson. Rationales often describe how a concept is continued later in the lesson or unit.



- What do the values of A, B, and C represent?
  A is the cost of each ride, B is the cost of each game, and C is the total amount of money Dion has to spend.
- What do the variables r and g represent?
  The variable r represents the number of rides he goes on, and g represents the number of games he plays.
- How is this form of equation helpful for this context?
  Answers will vary. Possible student response: This form allows us to quickly determine the relationship between the variables and to try out different combinations of values to see if they meet the given constraint.

Guiding Student Thinking

Facilitate academic conversation with students as you point out that in this situation, A, B, and C are whole numbers whose values are derived from the context. Ask students if the values of A, B, and C always have to be whole numbers. Allow students in discussion groups to critique the reasoning of one another as they grapple with this question. It is important that students do not develop misconceptions about limitations on the values of A, B, and C. While the values of A, B, and C in the standard form of a linear equation were defined to be integers in some older textbooks, this is an unnecessary restriction on the parameters. As long as A, B, and C are real numbers, any equation of the form $Ax + By = C$ is linear. For example, a graph of solutions to the equation $\sqrt{2}x + \sqrt{3}x = \pi$ also lie on a line. Although equations like this example are beyond the scope of Algebra 1, they are valid equations of lines expressed in standard form.

- Distribute Handout 1.9.B and allow students to complete it in pairs.
- Debrief by posting the answers to the handout and allowing student pairs to ask questions, discuss discrepancies with other pairs, and revise as needed.

Classroom Ideas

Consider including the new term *standard form* on your interactive word wall along with the definition $Ax + By = C$. This will support students as they connect the multiple representations of linear equations throughout the course.

Summarizing the Task

Guide students through summarizing the key concepts of the lesson by asking the following questions:

Next, invite them to mark the estimated balance point from their card stock triangle on the graph paper. Consider directing students to place the longer leg along the horizontal axis to keep students' work consistent with each other, as well as to match the examples and calculations that appear in this lesson.

Meeting Learners' Needs

For students who need an additional challenge, have them create a triangle with the dimensions of their own choice. Students who accept this challenge, will have equations and graphs that vary from the lesson. Including triangles of different dimensions allows students to make connections among multiple representations as they share and synthesize their findings.

- Now that students have drawn a triangle on their graph paper, invite them to generate ideas about how the centroid of a triangle might relate to its vertices and sides.
- Listen to students' suggestions. Introduce the terms *median* and *midpoint* during the discussion; if any students suggest cutting the triangle "in half," that is a good opportunity to discuss what "cutting in half" would mean for a triangle and the midpoint of a line segment. Display an image of a triangle and identify one of its medians. Tell students the midpoint divides a segment in half and that the median of a triangle is a line segment from one vertex that extends to the opposite side's midpoint.

Instructional Rationale

To facilitate academic conversation, it may be helpful to create an anchor chart that describes the terms *median* and *midpoint* and display it throughout this lesson. Encourage students to draw the midpoint and medians on a sketch of a triangle to support their understanding in this lesson.

- In order to encourage students' thinking about medians, ask the following questions:
  - How might you locate the midpoint of one side of a triangle?
    We could measure its length and mark the halfway point along that side. We could also fold the side onto itself so the endpoints line up.

Pre-AP Algebra 1 Assessments for Learning

Pre-AP Algebra 1 assessments function as components of the teaching and learning cycle. Progress is not measured by performance on any single assessment. Rather, Pre-AP Algebra 1 offers a place to practice, to grow, and to recognize that learning takes time.

LEARNING CHECKPOINTS

Based on the Pre-AP Algebra 1 Course Framework, the learning checkpoints require students to examine data, models, diagrams, and short texts—set in authentic contexts—in order to respond to a targeted set of questions that measure students' application of the key concepts and skills from the unit. All eight learning checkpoints are automatically scored, with results provided through feedback reports that contain explanations of all questions and answers as well as individual and class views for educators. Teachers also have access to assessment summaries on Pre-AP Classroom, which provide more insight into the question sets and targeted learning objectives for each assessment event.

The following tables provide a synopsis of key elements of the Pre-AP Algebra 1 learning checkpoints.

Format	Two learning checkpoints per unit Digitally administered with automated scoring and reporting Questions target both concepts and skills from the course framework
Time Allocated	Designed for one 45-minute class period per assessment
Number of Questions	10–12 questions per assessment including four-option multiple choice and technology-enhanced questions.

Domains Assessed	
Learning Objectives	Learning objectives within each key concept from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus assessed regularly across all eight learning checkpoints: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

Question Styles	Question sets consist of two or three questions that focus on a single stimulus or group of related stimuli, such as diagrams, graphs, or tables. Questions embed mathematical concepts in real-world contexts. *Please see page 66 for a sample question set that illustrates the types of questions included in Pre-AP learning checkpoints and the question bank.*

PERFORMANCE TASKS

Each unit includes one performance-based assessment designed to evaluate the depth of student understanding of key concepts and skills that are not easily assessed in a multiple-choice format.

These tasks, developed for students across a broad range of readiness levels, are accessible while still providing sufficient challenge and the opportunity to practice the analytical skills that will be required in AP mathematics courses and for college and career readiness. Teachers participating in the official Pre-AP Program will receive access to online learning modules to support them in evaluating student work for each performance task.

Format	One performance task per unit May be administered online or in print ▪ If administered online, then a score report is available. Educator scored using scoring guidelines
Time Allocated	Approximately 45 minutes or as indicated
Number of Questions	An open-response task with multiple parts

Domains Assessed	
Key Concepts	Key concepts and prioritized learning objectives from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

PRACTICE PERFORMANCE TASKS

One or more practice performance tasks in each unit provide students with the opportunity to practice applying skills and knowledge in a context similar to a performance task, but in a more scaffolded environment. These tasks include strategies for adapting instruction based on student performance and ideas for modifying or extending tasks based on students' needs.

SAMPLE PERFORMANCE TASK AND SCORING GUIDELINES

The following task and set of scoring guidelines are representative of what students and educators will encounter on the performance tasks. (The example below includes a portion of a practice performance task in Unit 1.)

PRACTICE PERFORMANCE TASK

Measuring the Wind Speed of a Hurricane

LEARNING OBJECTIVES

1.1.1 Determine whether a relationship is linear by identifying the constant rate of change.

1.1.4 Use function notation to determine inputs and outputs of functions.

1.1.5 Construct and apply linear functions to solve problems given mathematical or applied contexts.

1.2.1 Identify and interpret key characteristics of linear functions and their graphs.

AREAS OF FOCUS

- Greater Authenticity of Applications and Modeling
- Engagement in Mathematical Argumentation

SUGGESTED TIMING

~90 minutes

HANDOUTS

- Unit 1 Practice Performance Task: Measuring the Wind Speed of a Hurricane
- Unit 1 Practice Performance Task: Additional Practice Resources

PRACTICE PERFORMANCE TASK DESCRIPTION

In this practice performance task, students synthesize their learning from previous lessons to solve problems in a real-world context of meteorological phenomena. Students calculate and interpret the rate of change of a relationship using two ordered pairs, write a linear function that can be used to model the relationship, and use the function to complete problems about the quantities involved. The context for this lesson is the relationship between barometric pressure and wind speed during a hurricane.

Measuring the Wind Speed of a Hurricane

Animals, especially birds and fish, are highly sensitive to changes in atmospheric (barometric) pressure. These pressure changes can trigger instinctive behaviors in these animals that can alert us to incoming storms. Meteorologists have a more scientific method to predict weather patterns. They use devices called barometers to measure atmospheric pressure in units called millibars (mb). Meteorologists have found that by analyzing barometric pressure they can predict wind speed. The table below shows data collected from different hurricanes.

Barometric Pressure (mb)	Wind Speed (km/h)
1,000	100
980	140
960	180
940	220

(a) Using the table, find the rate of change to determine if the relationship between the barometric pressures and wind speeds represented here is linear. Explain your reasoning.

(b) Write a function, f, that you could use to predict the wind speed in kilometers per hour (km/h) for any barometric pressure, x, in mb.

(c) Evaluate $f(991.5)$. What is the meaning of the input 991.5 and its corresponding output in this context?

(d) Explain the meanings of the initial value and the rate of change for this context. Discuss the reasonableness of these values in the real world.

SCORING STUDENT WORK

Whether you decide to have students score their own solutions, have students score their classmates' solutions, or score the solutions yourself, use the practice performance task results to inform further instruction.

Measuring the Wind Speed of a Hurricane

Animals, especially birds and fish, are highly sensitive to changes in atmospheric (barometric) pressure. These pressure changes can trigger instinctive behaviors in these animals that can alert us to incoming storms. Meteorologists have a more scientific method to predict weather patterns. They use devices called barometers to measure atmospheric pressure in units called millibars (mb). Meteorologists have found that by analyzing barometric pressure they can predict wind speed. The table below shows data collected from different hurricanes.

Barometric Pressure (mb)	Wind Speed (km/h)
1,000	100
980	140
960	180
940	220

There are 11 possible points for this performance task.

Student Part (a)

(a) Using the table, find the rate of change to determine if the relationship between the barometric pressures and wind speeds represented here is linear. Explain your reasoning.

Sample Solution	Points Possible
I checked ordered pairs of the form (mb, km/h) between consecutive rows of the table and found the rate of change was constant. For example: Rate of change $= \dfrac{140-100}{980-1000} = -2$ Therefore, the relationship is linear because there is a constant rate of change, -2.	**3 points maximum** 1 point for writing and using the correct rate of change formula 1 point for correctly calculating the rate of change 1 point for correctly determining and explaining why that the relationship is linear

Student Part (b)

(b) Write a function, f, that you could use to predict the wind speed in kilometers per hour (km/h) for any barometric pressure, x, in mb.

Sample Solution	Points Possible
Let x be the barometric pressure and $f(x)$ be the wind speed. Using the point (1,000, 100) and the rate of change of -2, the initial value is b where $100 = -2(1{,}000) + b$ so $b = 2{,}100$. Therefore, the equation is $f(x) = -2x + 2{,}100$.	**3 points maximum** 1 point for correctly defining the variables 2 points for developing the correct equation (any equivalent form is acceptable)

Student Part (c)

(c) Evaluate $f(991.5)$. What is the meaning of the input 991.5 and its corresponding output in this context?

Sample Solution	Points Possible
$f(991.5) = -2(991.5) + 2{,}100 = 117$. The ordered pair (991.5, 117) indicates that when the barometric pressure is 991.5 mb, the wind speed is 117 km/h.	**2 points maximum** 1 point for correctly evaluating $f(991.5)$ 1 point for correctly interpreting the values (991.5, 117) in context

Student Part (d)

(d) Explain the meanings of the initial value and the rate of change for this context. Discuss the reasonableness of these values in the real world.

Sample Solution	Points Possible
The initial value is 2,100 because $f(0) = 2{,}100$. This means that if the barometric pressure is 0 mb, then the wind speed would be 2,100 km/h. However, in the context of real weather, this is not reasonable. A barometric pressure of 0 is much less than the lowest recorded barometric pressure, and 2,100 km/h is much greater than the highest recorded wind speed. The rate of change means that every decrease of 2 km/h in wind speed corresponds to a 1 mb increase in barometric pressure. Based on the data, this seems reasonable. *Scoring note*: Accept any correct interpretation for rate of change, for example, "The rate of change means that for every decrease of 1 mb in barometric pressure, there is an increase of 2 km/h in wind speed." *Scoring note*: If students refer to the rate of change as the slope of the equation, or the initial value as the y-intercept of a graph, do not penalize them on this practice performance task.	**3 points maximum** 1 point for accurately identifying and explaining the meaning of the initial value in context 1 point for accurately identifying and explaining the meaning of the rate of change in context 1 point for correctly explaining that the initial value is not reasonable and the rate of change is reasonable.

PROVIDING FEEDBACK ON STUDENT WORK

Because this is a practice performance task, consider sharing the scoring guidelines with students before you score the task. Reviewing the scoring guidelines provides students with an opportunity to see a complete response and self-assess their own progress and solution. After scoring your students' work, examine all responses to identify trends and inform further instruction. Look for topics where students consistently displayed mastery as well as topics that included frequent errors. Possible trends and suggested guidance for each part of the task follow, although your classroom results and trends may differ.

(a) Students likely understand that data must have a constant rate of change to be linear, but may err when calculating the rate of change from a table rather than from a graph. Students may invert the numerator and denominator in the formula, $\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$, or not subtract the ordered pairs in the same order, $\left(\frac{y_2 - y_1}{x_1 - x_2}\right)$. It may be helpful to refer them back to other problems where they calculated a rate of change from a tabular representation and then connect that method to the algebraic formula. The **Unit 1 Practice Performance Task: Additional Practice Resources** handout included with this practice performance task may also be helpful.

TEACHER NOTES AND REFLECTIONS

(b) Students will likely use the rate of change from part (a) to write the equation, but might switch the m and b values, or switch the input and output values when substituting an ordered pair into the equation to solve for b. Sign errors may also occur when students substitute values and simplify. Be sure that students substitute the correct m and b values into $f(x) = mx + b$ when they have found b. Direct students to write out each computational step using parentheses when they substitute values, before they begin simplifying.

TEACHER NOTES AND REFLECTIONS

(c) Students may have used y instead of $f(x)$ in the previous parts. Ask students to think aloud about their understanding of $f(x)$ and how it relates to y. Check to be sure students simplify correctly when working with the negative slope, both when multiplying and adding. It may be helpful to provide additional practice with evaluating two-step equations involving negative values.

TEACHER NOTES AND REFLECTIONS

(d) Students may solve $f(x) = 0$ rather than evaluate $f(0)$. Ask them whether the initial value represents when the input is 0 or the output is 0. It may help students to sketch a graph and label the axes with the quantity each one represents. Point to the axis labels as students read the coordinates of each point. If students need support interpreting the rate of change, provide a sentence stem such as: "For every 1 mb increase in barometric pressure, there is a ____km/h ______ in wind speed," or "For every 1 mb decrease in barometric pressure, there is a ____km/h ______ in wind speed." For additional practice with interpreting slope and rate of change in context, provide graphs for different contexts and ask partners to take turns verbalizing the meaning of each parameter.

TEACHER NOTES AND REFLECTIONS

Try to assure students that converting their score into a percentage does not provide an accurate measure of how they performed on the task. You can use the following suggested score interpretations with students to discuss their performance.

Points Received	How Students Should Interpret Their Score
10 or 11 points	"I know all of these algebraic concepts really well. This is top-level work. (A)"
7 to 9 points	"I know all of these algebraic concepts well, but I made a few mistakes. This is above-average work. (B)"
5 or 6 points	"I know some of these algebraic concepts well, but not all of them. This is average-level work. (C)"
2 to 4 points	"I know only a little bit about these algebraic concepts. This is below-average work. (D)"
0 or 1 point	"I don't know much about these algebraic concepts at all. This is not passing work. (F)"

QUESTION BANK

The Pre-AP question bank lets teachers select from hundreds of prewritten questions or create their own to build customized quizzes and assessments. These assessments allow students to showcase their mastery of the skills and content outlined in the Pre-AP Algebra 1 Course Framework. Each prewritten question in the question bank is indexed by unit and learning objectives, ensuring alignment with the course structure.

Teachers can tailor assessments by adding their own multiple-choice or free-response questions. These assessments can be administered online or printed for in-class use. When delivered online, Pre-AP Classroom generates a report with performance data for multiple-choice questions. Teachers can share their custom question bank assignments with other Pre-AP educators teaching the same course.

SAMPLE ASSESSMENT QUESTIONS

The following questions are representative of what students and educators will encounter on the learning checkpoints.

1. Jazmin runs a lemonade stand every Saturday. She sells each cup of lemonade for the same price. Which of the following statements describes how Jazmin can most accurately model her daily revenue?

   (A) She can use a linear function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, is constant.

   (B) She can use a quadratic function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, is linear.

   (C) She can use an exponential function to model her revenue because the growth factor, in terms of dollars earned per cup of lemonade sold, is constant.

   (D) She can use a piecewise linear function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, varies over the course of the day.

Assessment Focus

Question 1 assesses whether a student can identify an appropriate function to use for modeling a rate of change when given a real-world scenario.

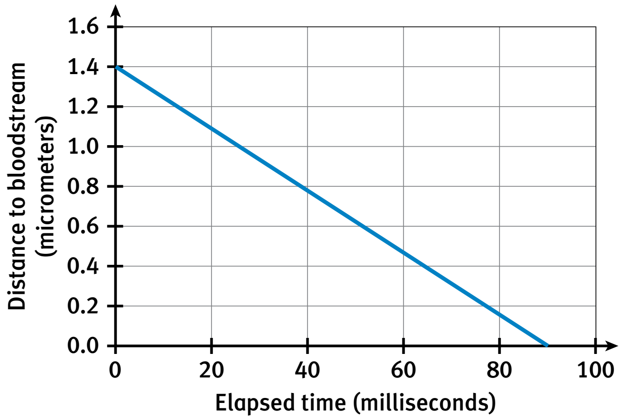
Correct Answer: A

Learning Objective:

1.1.1 Determine whether a relationship is linear by identifying the constant rate of change.

Area of Focus: Engagement in Mathematical Argumentation

2. In humans, oxygen molecules move from the lungs into the bloodstream, where they are carried throughout the body. The graph below shows the distance, in micrometers, an oxygen molecule is from the bloodstream and the elapsed time, in milliseconds, it took to move from the lungs into the bloodstream.



What is the value of the vertical axis intercept and what does it mean?

(A) 1.4 milliseconds is the time it took the oxygen molecule to move from the lungs into the bloodstream.

(B) 90 milliseconds is the time it took the oxygen molecule to move from the lungs into the bloodstream.

(C) 90 micrometers is the distance the oxygen molecule moved between the lungs and the bloodstream.

(D) 1.4 micrometers is the distance the oxygen molecule moved between the lungs and the bloodstream.

Assessment Focus

Question 2 assesses whether or not students can interpret the vertical axis intercept in context. They must apply the skill of interpreting information presented in a graph in order to solve the problem.

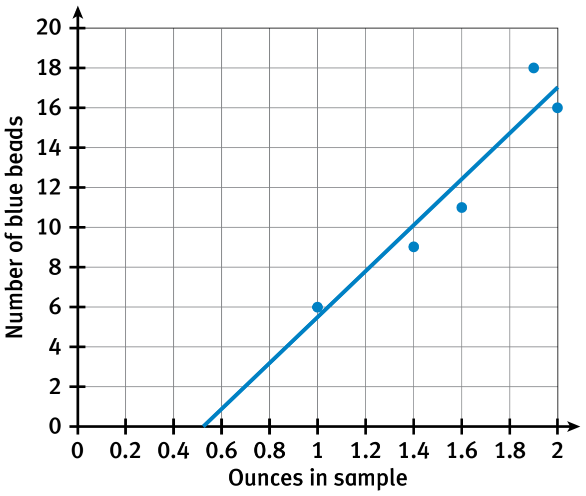
Correct Answer: D

Learning Objective:

1.2.1 Identify and interpret key characteristics of linear functions and their graphs.

Area of Focus: Greater Authenticity of Applications and Modeling

3. Eli is purchasing beads for an art project. The supply store sells the beads in bulk in multicolor assortments. Eli only wants to use blue beads for his project, but the store won't allow him to pick through the beads to select only blue ones. From the large bin of assorted beads, he takes five small samples of varying weight and counts the number of blue beads in each sample. The scatterplot represents his findings.



To determine an equation for a trend line, Eli uses two points that he thinks lie on the line: the number of blue beads for a sample that measures 0.6 ounces and the number of blue beads for a sample that measures 2 ounces. Which of the following is the correct equation of the trend line that Eli determined?

(A) $y=\left(\frac{16}{1.4}\right)x-\frac{41}{7}$

(B) $y=\left(\frac{15}{1.4}\right)x-\frac{38}{7}$

(C) $y=\left(\frac{1.4}{16}\right)x+\frac{673}{400}$

(D) $y=\left(\frac{1.4}{15}\right)x-\frac{1186}{75}$

Assessment Focus

Question 3 assesses whether or not students can determine the equation of the line of best fit. Students must also translate between information provided in the stimulus text and the scatterplot.

Correct Answer: A

Learning Objective:

4.3.1 Select an appropriate function model given a data set.

Area of Focus: Connections Among Multiple Representations

4. Laura is a general contractor who needs to order at least 110 sheets of plywood for a house she is building. There are two supply companies, Menureds and HouseCo, and neither one has enough plywood in stock for Laura's project. Menureds sells plywood in bundles of 3 sheets and they have 12 bundles available at $72 per bundle. HouseCo sells plywood in bundles of 4 sheets and they have 25 bundles available at $76 per bundle.

   Laura has budgeted up to $2,200 for plywood. Which combination of bundles gets Laura at least 110 sheets and and stays under budget?

   (A) 12 bundles from Menureds and 18 bundles from HouseCo
   (B) 10 bundles from Menureds and 20 bundles from HouseCo
   (C) 12 bundles from Menureds and 25 bundles from HouseCo
   (D) 5 bundles from Menureds and 24 bundles from HouseCo

Assessment Focus

Question 4 assesses whether or not students can represent and model a scenario involving a simple system of inequalities and interpret potential solutions.

Correct Answer: D

Learning Objective:

1.4.4 Construct and solve systems of linear inequalities that model mathematical or applied contexts.

Area of Focus: Greater Authenticity of Applications and Modeling

Offering Pre-AP Algebra 1

Schools can earn an official Pre-AP Algebra 1 course designation by completing the Algebra 1 Course Audit. All schools offering courses that have received a Pre-AP course designation will be listed in the Pre-AP Course Ledger, in a process like that used for listing authorized AP courses.

To support a successful implementation, teachers and students should have access to the full suite of Pre-AP Algebra 1 resources along with additional high quality instructional materials suitable for an Algebra 1 course. Pre-AP resources include:

- The Algebra 1 Course Framework
- Pre-AP Algebra 1 teacher and student resources
- AP Classroom resources and digital assessments
- Pre-AP Professional Learning

CURRICULAR COMMITMENTS

- The students and teacher have access to high-school-level Algebra 1 course materials in addition to Pre-AP instructional resources.
- The course is structured such that the content outlined in the four units of the Pre-AP Algebra 1 Course Framework forms the foundation of the course.
- The course provides opportunities for students to develop the skills related to the Pre-AP mathematics areas of focus:
  - greater authenticity in application and modeling
  - engagement in mathematical argumentation
  - connections among multiple representations
- The course uses instructional strategies to engage students in the Pre-AP shared principles of:
  - close observation and analysis
  - higher-order questioning
  - evidence-based writing
  - academic conversation
- The course provides frequent opportunities for students to engage in formative assessment, using Pre-AP or other assessment resources.

Accessing the Digital Materials

Pre-AP Classroom is the online application through which teachers and students can access Pre-AP instructional resources and assessments. The digital platform is similar to AP Classroom, the online system used for AP courses.

Pre-AP coordinators receive access to Pre-AP Classroom via an access code by calling customer care. Teachers receive access after the Pre-AP Course Audit process has been completed.

Once teachers have created course sections, students can enroll in them via an access code. When both teachers and students have access, teachers can share instructional resources with students, assign and score assessments, and complete online learning modules; students can view resources shared by the teacher, take assessments, and receive feedback reports to understand progress and growth.

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