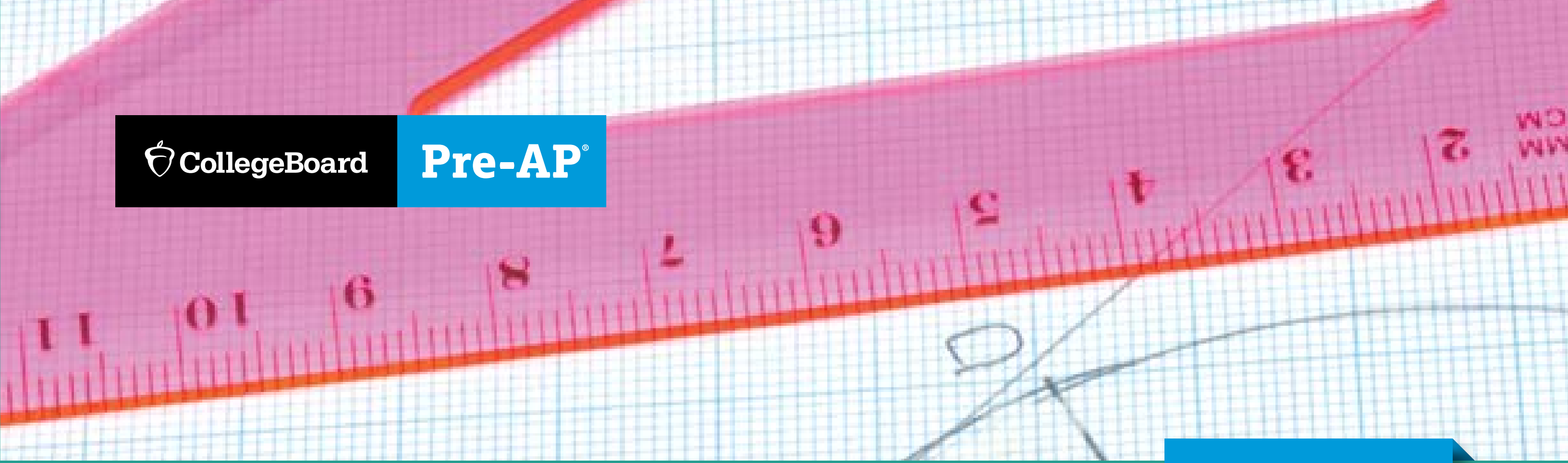


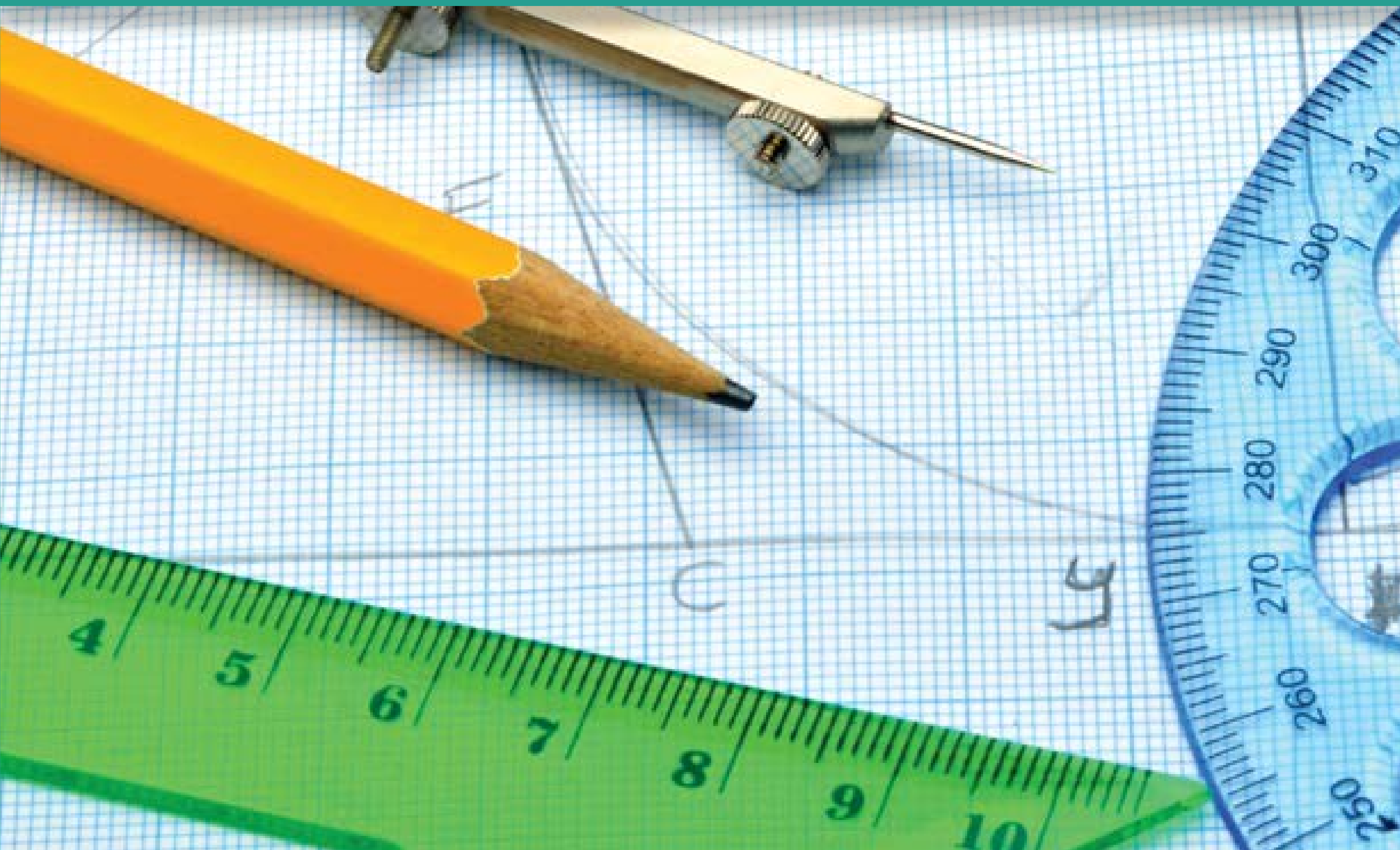
CollegeBoard Pre-AP®

Pre-AP® Geometry with Statistics

COURSE GUIDE

INCLUDES

- Approach to teaching and learning
- Course map
- Course framework
- Sample assessment questions



Pre-AP®
Geometry with Statistics

COURSE GUIDE

Please visit Pre-AP online at **preap.org** for more information and updates about the course and program features.

ABOUT COLLEGE BOARD

College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success—including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

For further information, visit **www.collegeboard.org**.

PRE-AP EQUITY AND ACCESS POLICY

College Board believes that all students deserve engaging, relevant, and challenging grade-level coursework. Access to this type of coursework increases opportunities for all students, including groups that have been traditionally underrepresented in AP and college classrooms. Therefore, the Pre-AP program is dedicated to collaborating with educators across the country to ensure all students have the supports to succeed in appropriately challenging classroom experiences that allow students to learn and grow. It is only through a sustained commitment to equitable preparation, access, and support that true excellence can be achieved for all students, and the Pre-AP course designation requires this commitment.

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Contents

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About Pre-AP

Introduction to Pre-AP

Every student deserves classroom opportunities to learn, grow, and succeed. College Board developed Pre-AP® to deliver on this simple premise. Pre-AP courses are designed to support all students across varying levels of readiness. They are not honors or advanced courses.

Participation in Pre-AP courses allows students to slow down and focus on the most essential and relevant concepts and skills. Students have frequent opportunities to engage deeply with texts, sources, and data as well as compelling higher-order questions and problems. Across Pre-AP courses, students experience shared instructional practices and routines that help them develop and strengthen the important critical thinking skills they will need to employ in high school, college, and life. Students and teachers can see progress and opportunities for growth through varied classroom assessments that provide clear and meaningful feedback at key checkpoints throughout each course.

DEVELOPING THE PRE-AP COURSES

Pre-AP courses are carefully developed in partnership with experienced educators, including middle school, high school, and college faculty. Pre-AP educator committees work closely with College Board to ensure that the course resources define, illustrate, and measure grade-level-appropriate learning in a clear, accessible, and engaging way. College Board also gathers feedback from a variety of stakeholders, including Pre-AP partner schools from across the nation who have participated in multiyear pilots of select courses. Data and feedback from partner schools, educator committees, and advisory panels are carefully considered to ensure that Pre-AP courses provide all students with grade-level-appropriate learning experiences that place them on a path to college and career readiness.

PRE-AP PROGRAM COMMITMENTS

The Pre-AP Program asks participating schools to make four commitments:

1. **Pre-AP for All:** Pre-AP frameworks and assessments serve as the foundation for all sections of the course at the school.
2. **Course Frameworks:** Teachers align their classroom instruction with the Pre-AP course frameworks.
    - Schools commit to provide the core resources to ensure Pre-AP teachers and students have the materials they need to engage in the course.

3. **Assessments:** Teachers administer at least one learning checkpoint per unit on Pre-AP Classroom and four performance tasks.
4. **Professional Learning:** Teachers complete the foundational professional learning (Online Foundational Modules or Pre-AP Summer Institute) and at least one online performance task scoring module. The current Pre-AP coordinator completes the Pre-AP Coordinator Online Module.

PRE-AP EDUCATOR NETWORK

Similar to the way in which teachers of Advanced Placement® (AP®) courses can become more deeply involved in the program by becoming AP Readers or workshop consultants, Pre-AP teachers also have opportunities to become active in their educator network. Each year, College Board expands and strengthens the Pre-AP National Faculty—the team of educators who facilitate Pre-AP Professional Learning Workshops. Pre-AP teachers can also become curriculum and assessment contributors by working with College Board to design, review, or pilot the course resources.

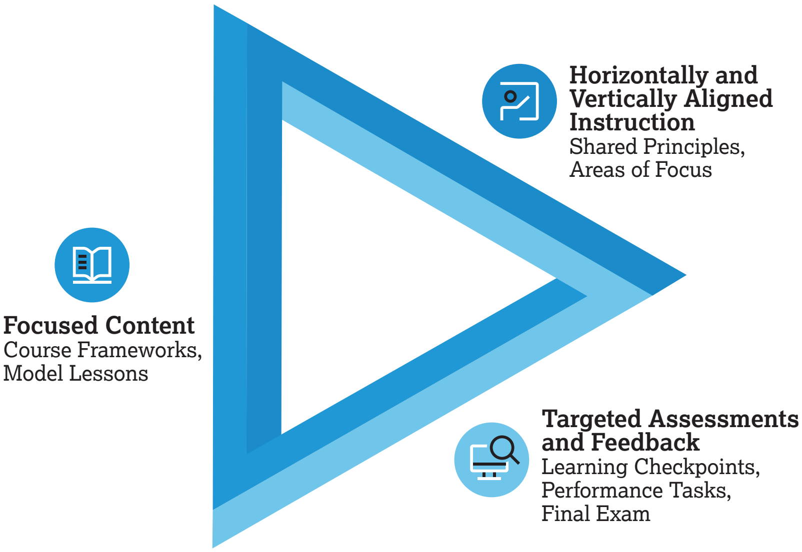
HOW TO GET INVOLVED

Schools and districts interested in learning more about participating in Pre-AP should visit **preap.org/join** or contact us at **preap@collegeboard.org**.

Teachers interested in becoming members of Pre-AP National Faculty or participating in content development should visit **preap.org/national-faculty** or contact us at **preap@collegeboard.org**.

Pre-AP Approach to Teaching and Learning

Pre-AP courses invite all students to learn, grow, and succeed through focused content, horizontally and vertically aligned instruction, and targeted assessments for learning. The Pre-AP approach to teaching and learning, as described below, is not overly complex, yet the combined strength results in powerful and lasting benefits for both teachers and students. This is our theory of action.



FOCUSED CONTENT

Pre-AP courses focus deeply on a limited number of concepts and skills with the broadest relevance for high school coursework and college and career success. The course framework serves as the foundation of the course and defines these prioritized concepts and skills. Pre-AP model lessons and assessments are based directly on this focused framework. The course design provides students and teachers with intentional permission to slow down and focus.

HORIZONTALLY AND VERTICALLY ALIGNED INSTRUCTION

Shared principles cut across all Pre-AP courses and disciplines. Each course is also aligned to discipline-specific areas of focus that prioritize the critical reasoning skills and practices central to that discipline.

SHARED PRINCIPLES

All Pre-AP courses share the following set of research-supported instructional principles. Classrooms that regularly focus on these cross-disciplinary principles allow students to effectively extend their content knowledge while strengthening their critical thinking skills. When students are enrolled in multiple Pre-AP courses, the horizontal alignment of the shared principles provides students and teachers across disciplines with a shared language for their learning and investigation, and multiple opportunities to practice and grow. The critical reasoning and problem-solving tools students develop through these shared principles are highly valued in college coursework and in the workplace.

Close Observation and Analysis

Students are provided time to carefully observe one data set, text image, performance piece, or problem before being asked to explain, analyze, or evaluate. This creates a safe entry point to simply express what they notice and what they wonder. It also encourages students to slow down and capture relevant details with intentionality to support more meaningful analysis, rather than rush to completion at the expense of understanding.

Higher-Order Questioning

Students engage with questions designed to encourage thinking that is elevated beyond simple memorization and recall. Higher-order questions require students to make predictions, synthesize, evaluate, and compare. As students grapple with these questions, they learn that being inquisitive promotes extended thinking and leads to deeper understanding.

Evidence-Based Writing

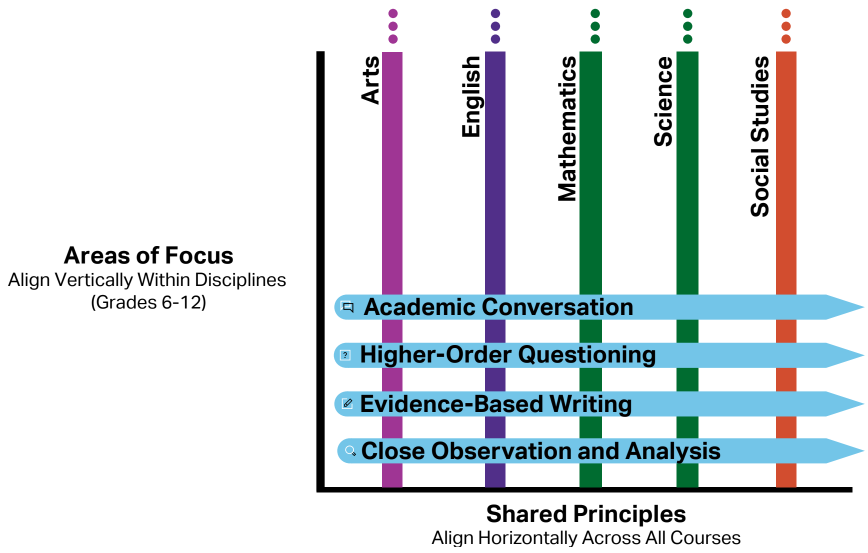
With strategic support, students frequently engage in writing coherent arguments from relevant and valid sources of evidence. Pre-AP courses embrace a purposeful and scaffolded approach to writing that begins with a focus on precise and effective sentences before progressing to longer forms of writing.

Academic Conversation

Through peer-to-peer dialogue, students' ideas are explored, challenged, and refined. As students engage in academic conversation, they come to see the value in being open to new ideas and modifying their own ideas based on new information. Students grow as they frequently practice this type of respectful dialogue and critique and learn to recognize that all voices, including their own, deserve to be heard.

AREAS OF FOCUS

The areas of focus are discipline-specific reasoning skills that students develop and leverage as they engage with content. Whereas the shared principles promote horizontal alignment across disciplines, the areas of focus provide vertical alignment within a discipline, giving students the opportunity to strengthen and deepen their work with these skills in subsequent courses in the same discipline.



For information about the Pre-AP mathematics areas of focus, see page 13.

TARGETED ASSESSMENTS FOR LEARNING

Pre-AP courses include strategically designed classroom assessments that serve as tools for understanding progress and identifying areas that need more support. The assessments provide frequent and meaningful feedback for both teachers and students across each unit of the course and for the course as a whole. For more information about assessments in Pre-AP Geometry with Statistics, see page 58.

Pre-AP Professional Learning

As part of the program commitments, Pre-AP teachers agree to engage in two professional learning opportunities:

1. The first commitment is designed to help prepare teachers to teach their specific course. There are two options to meet this commitment: the Pre-AP Summer Institute (Pre-APSI) and the Online Foundational Modules. Both options provide continuing education units to educators upon completion.
   - The Pre-AP Summer Institute provides a collaborative experience that empowers participants to prepare and plan for their Pre-AP course. While attending, teachers engage with Pre-AP course frameworks, shared principles, areas of focus, and sample model lessons. Participants are given supportive planning time where they work with peers to begin building their Pre-AP course plan.
   - Online Foundational Modules are available to all teachers of Pre-AP courses. In their 12- to 20-hour asynchronous course, teachers explore course materials and experience model lessons from the student's point of view. They also begin building their Pre-AP course plan.
2. The second professional learning opportunity helps teachers prepare for the performance tasks. As part of this commitment, teachers agree to complete at least one online performance task scoring module. Online scoring modules offer guidance and practice applying scoring guidelines and examining student work. Teachers may complete the modules independently or with teachers of the same course in their school's professional learning communities.

About Pre-AP Geometry with Statistics

Introduction to Pre-AP Geometry with Statistics

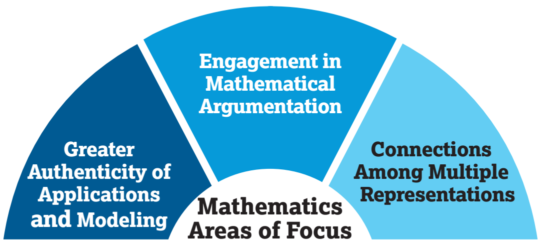
Pre-AP Geometry with Statistics is designed to provide students with a meaningful conceptual bridge between algebra and geometry to deepen their understanding of mathematics. Students often struggle to see the connections among their mathematics courses. In this course, students are expected to use the mathematical knowledge and skills they have developed previously to problem solve across the domains of algebra, geometry, and statistics.

Rather than seeking to cover all topics traditionally included in a standard geometry or introductory statistics textbook, this course focuses on the foundational geometric and statistical knowledge and skills that matter most for college and career readiness. The Pre-AP Geometry with Statistics Course Framework highlights how to guide students to connect core ideas within and across the units of the course, promoting a coherent understanding of measurement.

The components of this course have been crafted to prepare not only the next generation of mathematicians, scientists, programmers, statisticians, and engineers, but also a broader base of mathematically informed citizens who are well equipped to respond to the array of mathematics-related issues that impact our lives at the personal, local, and global levels.

PRE-AP MATHEMATICS AREAS OF FOCUS

The Pre-AP mathematics areas of focus, shown below, are mathematical practices that students develop and leverage as they engage with content. They were identified through educator feedback and research about where students and teachers need the most curriculum support. These areas of focus are vertically aligned to the mathematical practices embedded in other mathematics courses in high school, including AP, and in college, giving students multiple opportunities to strengthen and deepen their work with these skills throughout their educational career. They also support and align to the AP Calculus Mathematical Practices, the AP Statistics Course Skills, and the mathematical practices listed in various state standards.



Greater Authenticity of Applications and Modeling

Students create and use mathematical models to understand and explain authentic scenarios.

Mathematical modeling is a process that helps people analyze and explain the world. In Pre-AP Geometry with Statistics, students explore real-world contexts where mathematics can be used to make sense of a situation. They engage in the modeling process by making choices about what aspects of the situation to model, assessing how well the model represents the available data, drawing conclusions from their model, justifying decisions they make through the process, and identifying what the model helps clarify and what it does not.

In addition to mathematical modeling, Pre-AP Geometry with Statistics students engage in mathematics through authentic applications. Applications are similar to modeling problems in that they are drawn from real-world phenomena, but they differ because the applications dictate the appropriate mathematics to use to solve the problem. Pre-AP Geometry with Statistics balances these two types of real-world tasks.

Engagement in Mathematical Argumentation

Students use evidence to craft mathematical conjectures and prove or disprove them.

Reasoning and proof lie at the heart of the discipline of mathematics. Mathematics is both a way of thinking and a set of tools for solving problems. Pre-AP Geometry with Statistics students gain proficiency in deductively reasoning with axioms and theorems to reach logical conclusions. Students also develop skills in using statistical and probabilistic reasoning to make sense of data and craft assertions using data as evidence and support. Students learn how to quantify chance and make inferences about populations. Through these two different types of mathematical argumentation, students learn how to be critical of their own reasoning and the reasoning of others.

Connections Among Multiple Representations

Students represent mathematical concepts in a variety of forms and move fluently among the forms.

Pre-AP Geometry with Statistics students explore how to weave together multiple representations of geometric and statistics concepts. Every mathematical representation illuminates certain characteristics of a concept while also obscuring other aspects. Often, geometric reasoning is used to make sense of algebraic calculations. Likewise, algebraic techniques can be used to solve problems involving geometry. Patterns in data can emerge by depicting the data visually. Statistical calculations are important and valuable, but they make more sense to students when they are conceptually grounded in and related to graphical representations of data. With experience that continues to develop in Pre-AP Geometry with Statistics, students become equipped with a nuanced understanding of which representations best serve a particular purpose.

PRE-AP GEOMETRY WITH STATISTICS AND CAREER READINESS

The Pre-AP Geometry with Statistics course resources are designed to expose students to a wide range of career opportunities that depend on geometry and statistics knowledge and skills. Examples include not only field-specific specialty careers such as mathematicians or statisticians, but also other endeavors where geometry and statistics knowledge is relevant, such as architects, carpenters, engineers, mechanics, actuaries, and programmers.

Career clusters that involve geometry and statistics, along with examples of careers in mathematics or related to mathematics, are provided below and on the following page. Teachers should consider discussing these with students throughout the year to promote motivation and engagement.

Career Clusters Involving Mathematics
agriculture, food, and natural resources architecture and construction arts, A/V technology, and communications business management and administration finance government and public administration health science information technology manufacturing marketing STEM (science, technology, engineering, and math) transportation, distribution, and logistics

Examples of Geometry Related Careers	Examples of Statistics Related Careers
animator architect cartographer drafter mathematician mathematics teacher professor programmer surveyor	drafter economist financial analyst mathematics teacher meteorologist professor programmer research analyst statistician

Source for Career Clusters: "Advanced Placement and Career and Technical Education: Working Together." Advance CTE and the College Board. October 2018. **https://careertech.org/resource/ap-cte-working-together**.

For more information about careers that involve mathematics, teachers and students can visit and explore the College Board's Big Future resources: **https://bigfuture.collegeboard.org/majors/math-statistics-mathematics**.

SUMMARY OF RESOURCES AND SUPPORTS

Teachers are strongly encouraged to take advantage of the full set of resources and supports for Pre-AP Geometry with Statistics, which is summarized below. Some of these resources are part of the Pre-AP Program commitments that lead to Pre-AP Course Designation. To learn more about the commitments for course designation, see details below and on page 75.

COURSE FRAMEWORK

Included in this guide as well as in the *Pre-AP Geometry with Statistics Teacher Resources*, the framework defines what students should know and be able to do by the end of the course. It serves as an anchor for model lessons and assessments, and it is the primary resource needed to plan the course. **Teachers commit to aligning their classroom instruction with the course framework**. *For more details see page 22.*

MODEL LESSONS

Teacher resources, available in print and online, include a robust set of model lessons that demonstrate how to translate the course framework, shared principles, and areas of focus into daily instruction. **Use of the model lessons is encouraged**. *For more details see page 56.*

LEARNING CHECKPOINTS

Accessed through Pre-AP Classroom (the Pre-AP digital platform), these short formative assessments provide insight into student progress. They are automatically scored and include multiple-choice and technology-enhanced items with rationales that explain correct and incorrect answers. **Teachers commit to administering one learning checkpoint per unit**. *For more details see page 58.*

PERFORMANCE TASKS

Available in the printed teacher resources as well as on Pre-AP Classroom, performance tasks allow students to demonstrate their learning through extended problem-solving, writing, analysis, and/or reasoning tasks. Scoring guidelines are provided to inform teacher scoring, with additional practice and feedback suggestions available in online modules on Pre-AP Classroom. **Teachers commit to using each unit's performance task**. *For more details see page 60.*

PRACTICE PERFORMANCE TASKS

Available in the student resources, with supporting materials in the teacher resources, these tasks provide an opportunity for students to practice applying skills and knowledge as they would in a performance task, but in a more scaffolded environment. **Use of the practice performance tasks is encouraged**. *For more details see page 60.*

FINAL EXAM

Accessed through Pre-AP Classroom, the final exam serves as a classroom-based, summative assessment designed to measure students' success in learning and applying the knowledge and skills articulated in the course framework. **Administration of the final exam is encouraged**. *For more details see page 70.*

PROFESSIONAL LEARNING

Both the Pre-AP Summer Institute (Pre-APSI) and the Online Foundational Modules support teachers in preparing and planning to teach their Pre-AP course. **All Pre-AP teachers make a commitment to either attend the Pre-APSI (in person or virtually) or complete the Online Foundational Modules. In addition, teachers agree to complete at least one Online Performance Task Scoring module**. *For more details see page 9.*

Course Map

PLAN

The course map shows how components are positioned throughout the course. As the map indicates, the course is designed to be taught over 140 class periods (based on 45-minute class periods), for a total of 28 weeks.

Model lessons are included for approximately 50% of the total instructional time, with the percentage varying by unit. Each unit is divided into key concepts.

TEACH

The model lessons demonstrate how the Pre-AP shared principles and mathematics areas of focus come to life in the classroom.

Shared Principles
Close observation and analysis
Higher-order questioning
Evidence-based writing
Academic conversation

Areas of Focus
Greater authenticity of applications and modeling
Engagement in mathematical argumentation
Connections among multiple representations

ASSESS AND REFLECT

Each unit includes two learning checkpoints and a performance task. These formative assessments are designed to provide meaningful feedback for both teachers and students.

Note: The final exam, offered during a six-week window in the spring, is not represented on the map.

UNIT 1 Measurement in Data

~35 Class Periods

Pre-AP model lessons provided for 100% of instructional time in this unit

KEY CONCEPT 1.1
The Shape of Data

Learning Checkpoint 1

KEY CONCEPT 1.2
Chance Events

KEY CONCEPT 1.3
Inferences from Data

Learning Checkpoint 2

Performance Task for Unit 1

UNIT 2 Tools and Techniques of Geometric Measurement

~35 Class Periods

Pre-AP model lessons provided for approximately 50% of instructional time in this unit

KEY CONCEPT 2.1
Measurement in Geometry

Learning Checkpoint 1

KEY CONCEPT 2.2
Parallel and Perpendicular Lines

KEY CONCEPT 2.3
Measurement in Right Triangles

Learning Checkpoint 2

Performance Task for Unit 2

UNIT 3 Measurement in Congruent and Similar Figures

~35 Class Periods

Pre-AP model lessons provided for approximately 30% of instructional time in this unit

KEY CONCEPT 3.1
Transformations of Points in a Plane

KEY CONCEPT 3.2
Congruent and Similar Polygons

Learning Checkpoint 1

KEY CONCEPT 3.3
Measurement of Lengths and Angles in Circles

Learning Checkpoint 2

Performance Task for Unit 3

UNIT 4 Measurement in Two and Three Dimensions

~35 Class Periods

Pre-AP model lessons provided for approximately 10% of instructional time in this unit

KEY CONCEPT 4.1
Area as a Two-Dimensional Measurement

KEY CONCEPT 4.2
Learning Objectives 4.2.1–4.2.4
Volume as a Three-Dimensional Measurement

Learning Checkpoint 1

KEY CONCEPT 4.2 (*continued*)
Learning Objectives 4.2.5–4.2.7
Volume as a Three-Dimensional Measurement

KEY CONCEPT 4.3
Measurements of Spheres

Learning Checkpoint 2

Performance Task for Unit 4

Pre-AP Geometry with Statistics Course Framework

INTRODUCTION

Based on the Understanding by Design® (Wiggins and McTighe) model, the Pre-AP Geometry with Statistics Course Framework is back mapped from AP expectations and aligned to essential grade-level expectations. The course framework serves as a teacher's blueprint for the Pre-AP Geometry with Statistics instructional resources and assessments.

The course framework was designed to meet the following criteria:

- **Focused:** The framework provides a deep focus on a limited number of concepts and skills that have the broadest relevance for later high school, college, and career success.
- **Measurable:** The framework's learning objectives are observable and measurable statements about the knowledge and skills students should develop in the course.
- **Manageable:** The framework is manageable for a full year of instruction, fosters the ability to explore concepts in depth, and enables room for additional local or state standards to be addressed where appropriate.
- **Accessible:** The framework's learning objectives are designed to provide all students, across varying levels of readiness, with opportunities to learn, grow, and succeed.

COURSE FRAMEWORK COMPONENTS

The Pre-AP Geometry with Statistics Course Framework includes the following components:

Big Ideas

The big ideas are recurring themes that allow students to create meaningful connections between course concepts. Revisiting the big ideas throughout the course and applying them in a variety of contexts allows students to develop deeper conceptual understandings.

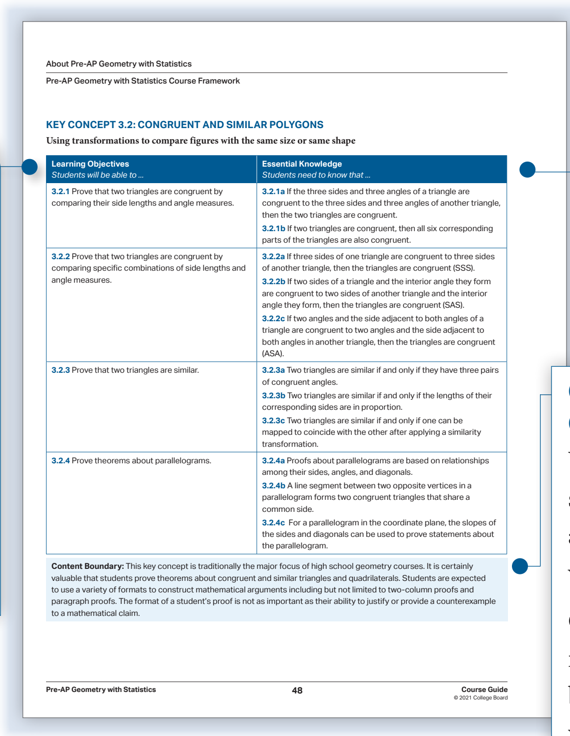
Enduring Understandings

Each unit focuses on a small set of enduring understandings. These are the long-term takeaways related to the big ideas that leave a lasting impression on students. Students build and earn these understandings over time by exploring and applying course content throughout the year.

Key Concepts

To support teacher planning and instruction, each unit is organized by key concepts. Each key concept includes relevant **learning objectives** and **essential knowledge statements** and may also include **content boundary and cross connection statements**. These are illustrated and defined below.

Learning Objectives:
These objectives define what a student needs to be able to do with essential knowledge to progress toward the enduring understandings. The learning objectives serve as actionable targets for instruction and assessment.



About Pre-AP Geometry with Statistics

Pre-AP Geometry with Statistics Course Framework

KEY CONCEPT 3.2: CONGRUENT AND SIMILAR POLYGONS

Using transformations to compare figures with the same size or same shape

Learning Objectives *Students will be able to ...*	Essential Knowledge *Students need to know that ...*
3.2.1 Prove that two triangles are congruent by comparing their side lengths and angle measures.	**3.2.1a** If the three sides and three angles of a triangle are congruent to the three sides and three angles of another triangle, then the two triangles are congruent. **3.2.1b** If two triangles are congruent, then all six corresponding parts of the triangles are also congruent.
3.2.2 Prove that two triangles are congruent by comparing specific combinations of side lengths and angle measures.	**3.2.2a** If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent (SSS). **3.2.2b** If two sides of a triangle and the interior angle they form are congruent to two sides of another triangle and the interior angle they form, then the triangles are congruent (SAS). **3.2.2c** If two angles and the side adjacent to both angles of a triangle are congruent to two angles and the side adjacent to both angles in another triangle, then the triangles are congruent (ASA).
3.2.3 Prove that two triangles are similar.	**3.2.3a** Two triangles are similar if and only if they have three pairs of congruent angles. **3.2.3b** Two triangles are similar if and only if the lengths of their corresponding sides are in proportion. **3.2.3c** Two triangles are similar if and only if one can be mapped to coincide with the other after applying a similarity transformation.
3.2.4 Prove theorems about parallelograms.	**3.2.4a** Proofs about parallelograms are based on relationships among their sides, angles, and diagonals. **3.2.4b** A line segment between two opposite vertices in a parallelogram forms two congruent triangles that share a common side. **3.2.4c** For a parallelogram in the coordinate plane, the slopes of the sides and diagonals can be used to prove statements about the parallelogram.

Content Boundary: This key concept is traditionally the major focus of high school geometry courses. It is certainly valuable that students prove theorems about congruent and similar triangles and quadrilaterals. Students are expected to use a variety of formats to construct mathematical arguments including but not limited to two-column proofs and paragraph proofs. The format of a student's proof is not as important as their ability to justify or provide a counterexample to a mathematical claim.

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Essential Knowledge Statements:
Each essential knowledge statement is linked to a learning objective. One or more essential knowledge statements describe the knowledge required to perform each learning objective.

Content Boundary and Cross Connection Statements:
When needed, content boundary statements provide additional clarity about the content and skills that lie within versus outside of the scope of this course.

Cross connection statements highlight important connections that should be made between key concepts within and across the units.

BIG IDEAS IN PRE-AP GEOMETRY WITH STATISTICS

While the Pre-AP Geometry with Statistics framework is organized into four core units of study, the content is grounded in three big ideas, which are cross-cutting concepts that build conceptual understanding and spiral throughout the course. Since these ideas cut across units, they serve as the underlying foundation for the enduring understandings, key concepts, and learning objectives that make up the focus of each unit. A deep and productive understanding in Pre-AP Geometry with Statistics relies on these three big ideas:

- **Measurement:** Measurement is the quantification of features of an object or a phenomenon. In geometry, measuring objects allows us to draw meaningful conclusions about those objects. Measurement provides relatable real-world applications in one, two, and three dimensions.
- **Transformation:** A transformation is a function, which means that it associates one set of objects with another. When a mathematical object is transformed, some of its measurements change while other measurements do not change. Congruence and similarity are defined through transformations, which puts the focus on measurements that are affected by transformations and those that are not. An understanding how data distributions are affected by transformations enhances the connections between probability and statistics.
- **Comparison and Composition:** Throughout mathematics, new and more complex concepts are understood in terms of simpler, previously explored concepts. In geometry, this mode of thinking allows for the deconstruction of two- and three-dimensional shapes for further investigation. This interpretation relies on the recognition that complex objects are composed of, and can be compared to, simpler objects. For statistics, this means using measures of center and spread to characterize complex data distributions.

OVERVIEW OF PRE-AP GEOMETRY WITH STATISTICS UNITS AND ENDURING UNDERSTANDINGS

Unit 1: Measurement in Data	Unit 2: Tools and Techniques of Geometric Measurement
▪ Statistics are numbers that summarize large data sets by reducing their complexity to a few key values that model their center and spread. ▪ Distributions are functions whose displays are used to analyze data sets. ▪ Probabilistic reasoning allows us to anticipate patterns in data. ▪ The method by which data are collected influences what can be said about the population from which the data were drawn, and how certain those statements are.	▪ A formal mathematical argument establishes new truths by logically combining previously known facts. ▪ Measuring features of geometric figures is the process of assigning numeric values to attributes of the figures, which allows the attributes to be compared. ▪ Pairs of lines in a plane that never intersect or that intersect at right angles have special geometric and algebraic properties. ▪ Right triangles are simple geometric shapes in which we can relate the measures of acute angles to ratios of their side lengths.
Unit 3: Measurement in Congruent and Similar Figures	**Unit 4: Measurement in Two and Three Dimensions**
▪ Transformations are functions that can affect the measurements of a geometric figure. ▪ Congruent figures have equal corresponding angle measures and equal distances between corresponding pairs of points. ▪ Similar figures have equal corresponding angle measurements, and the distances between corresponding pairs of points are proportional. ▪ The geometry of a circle is completely determined by its radius.	▪ The area of a figure depends on its height and its cross-sectional widths. ▪ The volume of a solid depends on its height and its cross-sectional areas. ▪ The geometry of a sphere is completely determined by its radius.

Unit 1: Measurement in Data

Suggested Timing: Approximately 7 weeks

This unit offers a sustained and focused examination of statistics and probability to support the development of students' quantitative literacy. Statistics and probability help us perform essential real-world tasks such as making informed choices, deciding between different policies, and weighing competing knowledge claims. While topics of statistics and probability are commonplace in high school geometry courses, students often have limited opportunities to engage in statistical and probabilistic reasoning and sense-making. To move students toward a sophisticated understanding of data, students are expected to think about data sets as distributions which are functions that associate data values with their frequency or their probability. This encourages students to connect their knowledge of functions to concepts of statistics and probability, creating a more complete understanding of mathematics. Throughout the unit, students generate their own data through surveys, experiments, and simulations that investigate some aspect of the real world. They engage in statistical calculations and probabilistic reasoning as methods of analysis to make sense of data and draw inferences about populations. Incorporating statistics and probability in the same course as geometry allows students to experience two distinct forms of argumentation: geometrical reasoning as drawing conclusions with certainty about an ideal mathematical world, and probabilistic reasoning as drawing less-than-certain conclusions about the real world. The conclusions of a probability argument are presented as ranges that have varying degrees of certainty.

ENDURING UNDERSTANDINGS

Students will understand that ...

- Statistics are numbers that summarize large data sets by reducing their complexity to a few key values that model their center and spread.
- Distributions are functions whose displays are used to analyze data sets.
- Probabilistic reasoning allows us to anticipate patterns in data.
- The method by which data are collected influences what can be said about the population from which the data were drawn, and how certain those statements are.

KEY CONCEPTS

- **1.1: The shape of data** – Identifying measures of center and spread to summarize and characterize a data distribution
- **1.2: Chance events** – Exploring patterns in random events to anticipate the likelihood of outcomes
- **1.3: Inferences from data** – Using probability and statistics to make claims about a population

KEY CONCEPT 1.1: THE SHAPE OF DATA

Identifying measures of center and spread to summarize and characterize a data distribution

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.1.1 Determine appropriate summary statistics for a quantitative data distribution.	**1.1.1a** A data distribution is a function whose input is each value in a data set and whose output is the corresponding frequency of that value. **1.1.1b** Summary statistics describe the important features of data distributions including identifying a typical value, also called the center of the data, and describing the clustering of the data around the typical value, also called the spread of the data. **1.1.1c** The mean and the median summarize a data distribution by identifying a typical value, or center, of the distribution. The mean and median have the same units as the values in the data distribution. **1.1.1d** The standard deviation, interquartile range, and range summarize a data distribution by quantifying the variability, or spread, of the data set. The standard deviation, interquartile range, and range have the same units as the values in the data distribution.
1.1.2 Create a graphical representation of a quantitative data set.	**1.1.2a** A boxplot summarizes a quantitative data set by partitioning its values into four groups, each consisting of the same number of data values. Boxplots are used to depict the spread of a distribution. **1.1.2b** A histogram summarizes a quantitative data set by partitioning its values into equal-width intervals and displaying bars whose heights indicate the frequency of values contained in each interval. Histograms are used to depict the shape of a distribution.
1.1.3 Analyze data distributions with respect to their centers.	**1.1.3a** The mean is the only point in the domain of a distribution where the sum of the deviations, or differences, between the mean and each point in the distribution is zero. **1.1.3b** The mean can be thought of as the center of mass of the data set. It is a weighted average that accounts for the number of data points that exists for every given value in the data set. **1.1.3c** Measures of center can be used to compare the typical values of the distributions. They provide useful information about whether one distribution is typically larger, smaller, or about the same as another distribution.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.1.4 Analyze data distributions with respect to their symmetry or direction of skew.	**1.1.4a** For symmetric distributions, such as the normal distribution, the proportion of data in any range to the left of the mean is equal to the proportion of data in the corresponding range to the right of the mean. **1.1.4b** Skew describes the asymmetry of a distribution. The direction of skew is indicated by the longer tail of data values in an asymmetric distribution. **1.1.4c** When a distribution is skewed, its mean and median will differ. The farther apart the mean and median are in a distribution, the more skewed the distribution will appear.
1.1.5 Analyze data distributions with respect to their variability.	**1.1.5a** Measures of variability quantify the typical spread of a data distribution. They are used to describe how similar the values of a data set are to each other. A distribution with low variability will have data values that are clustered at the center, so the distribution is well characterized by its measures of center. A distribution with high variability will have data values that are spread out from the center, so the distribution is less well characterized by its measures of center. **1.1.5b** The interquartile range is the length of the interval that contains the middle 50% of the values in a distribution. **1.1.5c** The total variation of a distribution can be measured by the sum of the squared deviations from the mean. The variance of a distribution is the average of the squared deviations from the mean. **1.1.5d** The standard deviation is the square root of the variance. The standard deviation can be interpreted as a typical distance of the data values from the mean.
1.1.6 Model a data distribution with a normal distribution.	**1.1.6a** The normal distribution is a model of a data distribution defined by its mean and standard deviation. The normal distribution is bell-shaped and symmetric about the mean. In a normal distribution, the frequency of data values tapers off at one standard deviation above or below the mean. **1.1.6b** When a normal distribution is used to model a data distribution, approximately 68% of the data values fall within one standard deviation of the mean. Approximately 95% of the data values fall within two standard deviations of the mean. Over 99% of the data values fall within three standard deviations of the mean. **1.1.6c** For normally distributed data, the mean and median are the same number, and they correspond to the mode, which is the value in the distribution with the highest frequency.

Content Boundary: In this unit, students are introduced to the normal distribution as a model for some data distributions, similar to how linear functions can be used to model some two-variable data sets. The normal distribution is often used to answer probabilistic questions. Those types of questions should be reserved for the lessons of Key Concept 1.2: Chance events.

Cross Connection: Students likely come to this course with a basic understanding of how to calculate some summary statistics, but with limited conceptual understanding about their meaning and utility. A goal of this unit is to expand students' understanding of measures of center and spread. The focus of the unit should be on using these measures to analyze data distributions.

KEY CONCEPT 1.2: CHANCE EVENTS

Exploring patterns in random events to anticipate the likelihood of outcomes

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.2.1 Create or analyze a data display for a categorical data set.	**1.2.1a** Venn diagrams and contingency tables are common displays of categorical data and are useful for answering questions about probability. **1.2.1b** The intersection of two categories is the set of elements common to both categories. **1.2.1c** The union of two categories is the set of elements found by combining all elements of both categories. **1.2.1d** For categorical data, variability is determined by comparing relative frequencies of categories.
1.2.2 Determine the probability of an event.	**1.2.2a** The sample space is the set of all outcomes of an experiment or random trial. An event is a subset of the sample space. **1.2.2b** Probabilities are numbers between 0 and 1 where 0 means there is no possibility that an event can occur, and 1 means the event is certain to occur. The probability of an event occurring can be described numerically as a ratio of the number of favorable outcomes to the number of total outcomes in a sample space. **1.2.2c** A probability distribution is a function that associates a probability with each possible value or interval of values for a random variable. The sum of the probabilities over all possible values of the independent variable must be 1.
1.2.3 Calculate relative frequencies, joint frequencies, marginal frequencies, or conditional probabilities for a categorical data set.	**1.2.3a** Relative frequencies are the number of times an event occurs divided by the total number of observations. They can be used to estimate probabilities of future events occurring. **1.2.3b** Joint frequencies are events that co-occur for two or more variables. They are the frequencies displayed in cells in a two-way contingency table. **1.2.3c** Marginal frequencies are events that summarize the frequencies across all levels of one variable while holding the second variable constant. They are the row totals and column totals in a two-way contingency table. **1.2.3d** The conditional probability of *B*, given *A* has already occurred, is the proportion of times *B* occurs when restricted to events only in *A*.
1.2.4 Determine if two events are independent.	**1.2.4a** Two events, *A* and *B*, are independent if the occurrence of *A* does not affect the probability of *B*. **1.2.4b** Two events, *A* and *B*, are independent if the probability of *A* and *B* occurring together is the product of their probabilities.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.2.5 Calculate the probability of a range of values of an independent variable, given a mean, standard deviation, and normal distribution.	**1.2.5a** The normal distribution can be used to model a probability distribution that is bell-shaped and symmetric about the mean. **1.2.5b** When a normal distribution is used as a model of a probability distribution, the probability of a data value occurring above the mean is 0.5 and the probability of a data value occurring below the mean is 0.5. **1.2.5c** When a normal distribution is used as a model of a probability distribution, the probability of data occurring within one standard deviation of the mean is approximately 0.68, the probability of data occurring within two standard deviations of the mean is approximately 0.95, and the probability of data occurring within three standard deviations of the mean is approximately 0.997. These proportions can be used to determine the probability of an event occurring in a population.

Content Boundary: Throughout the course framework, the terms *random variable* and *independent variable* are used interchangeably. These terms describe different aspects of the same variable. The term *random variable* describes the process by which the variable was sampled, and *independent variable* is used when the frequency or probability distribution of the variable is of interest.

Cross Connection: In this unit, students will explore how the normal distribution can be used to model a probability distribution. This is a slightly different application of the normal distribution than students used in the previous key concept. In those lessons, students were expected to answer questions about the percent of data, or expected percent of data, that occurred within certain ranges. In the lessons of this key concept, students are expected to answer questions about the probability that an event would occur within a given range.

Cross Connection: Students will likely have some understanding of probability and randomness from previous courses. It is important for students to understand that, mathematically, the term *random* means that the outcome of a single trial may not be known, although over repeated trials, the proportions of the different outcomes may be predictable.

KEY CONCEPT 1.3: INFERENCES FROM DATA

Using probability and statistics to make claims about a population

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.3.1 Distinguish between accuracy and precision as measures of statistical variability and statistical bias in measurements.	**1.3.1a** Accuracy is how close the measurements in a measurement process are to the true value being estimated. Accuracy is determined by comparing the center of a sample of measurements to the true value of the measure. **1.3.1b** Precision is how close the measurements in a measurement process are to one another. Precision is determined by examining the variability of a sample of measurements. **1.3.1c** Bias is the tendency of a measurement process to systematically overestimate or underestimate the true measure of a phenomenon. Bias is an indication of the inaccuracy of the measurement process.
1.3.2 Describe how the size of a sample impacts how well it represents the population from which it was drawn.	**1.3.2a** The law of large numbers states that the mean of the results obtained from a large number of trials will tend to become closer to the true value of the phenomenon being measured as more trials are performed. This means we can trust larger samples more than smaller ones. **1.3.2b** The law of large numbers assumes that there is no systematic error of measurement in the sample.
1.3.3 Design a method for gathering data that is appropriate for a given purpose.	**1.3.3a** An experiment is a method of gathering information about phenomena where the independent variable is manipulated by the researcher. **1.3.3b** An observational study is a method of gathering information about phenomena where the independent variable is not under the control of the researcher. **1.3.3c** A survey is a method of gathering information from a sample of people using a questionnaire.
1.3.4 Identify biases in sampling methods for experiments, observational studies, and surveys.	**1.3.4a** Experiments can be subject to systematic bias if the experiment does not sample from the population randomly and does not randomly assign sampling units to experimental and control conditions. **1.3.4b** Observational studies can be subject to sampling bias if the sampling unit being observed is not selected randomly. **1.3.4c** Surveys can be subject to bias from several factors, including sampling bias and response bias.

Content Boundary: A traditionally challenging concept is informally introduced in this key concept: the law of large numbers (Learning Objective 1.3.2). For this concept, students should not be expected to develop a complete understanding. A full understanding of the law of large numbers is beyond the scope of this course. If students go on to take a more advanced statistics course, such as AP Statistics, they will explore this concept more thoroughly.

Unit 2: Tools and Techniques of Geometric Measurement

Suggested Timing: Approximately 7 weeks

This unit introduces students to the basic objects of geometry and the tools used to explore these objects throughout the remainder of the course. The basic objects students investigate in this unit include lines, rays, segments, and angles. These figures serve as the building blocks of more complex objects that students explore in later units. Students continue to expand their understanding of measurement by developing techniques for quantifying and comparing the attributes of geometric objects. The tools they use to analyze objects include straightedges, compasses, rulers, protractors, dynamic geometry software, the coordinate plane, and right triangles. In addition, students use an informal understanding of transformations throughout the unit to justify whether two basic objects are congruent. They formalize transformations and define congruence and similarity through transformations in Unit 3. This unit culminates with an introduction to right triangle trigonometry, which integrates the tools and techniques of the unit into an investigation of new ways to express the relationship between angle measures and side lengths.

Throughout Units 2–4, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables, flowcharts, or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.

ENDURING UNDERSTANDINGS

Students will understand that …

- A formal mathematical argument establishes new truths by logically combining previously known facts.
- Measuring features of geometric figures is the process of assigning numeric values to attributes of the figures, which allows the attributes to be compared.
- Pairs of lines in a plane that never intersect or that intersect at right angles have special geometric and algebraic properties.
- Right triangles are simple geometric shapes in which we can relate the measures of acute angles to ratios of their side lengths.

KEY CONCEPTS

- **2.1: Measurement in geometry –** Using lengths, angles, and distance to describe and compare shapes
- **2.2: Parallel and perpendicular lines –** Determining if and how lines intersect to analyze spatial relationships in the real world
- **2.3: Measurement in right triangles –** Using the relationships between the side lengths and angle measures of right triangles to create new measurements

KEY CONCEPT 2.1: MEASUREMENT IN GEOMETRY

Using lengths, angles, and distance to describe and compare shapes

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.1.1 Describe and correctly label a line, ray, or line segment.	**2.1.1a** For any two distinct points in a plane, there is only one line that contains them. **2.1.1b** A line is straight, has no width, extends infinitely in two directions, and contains infinitely many points. A line can be named by a single lowercase letter, or it can be named by any two distinct points that lie on the line. **2.1.1c** A ray is a portion of a line that has a single endpoint and extends infinitely in one direction. A ray can be named by its endpoint and any other point on the ray, with its endpoint listed first. **2.1.1d** A line segment is a portion of a line between and including two endpoints. A line segment can be named by its two endpoints.
2.1.2 Describe and correctly label an angle.	**2.1.2a** An angle is a geometric figure formed when two lines, line segments, or rays share an endpoint. The point common to both lines, line segments, or rays is called the vertex of the angle. **2.1.2b** An angle can be named by its vertex. An angle can also be named using its vertex and the names of a nonvertex point that lies on each of its sides. For such angle names, the point that indicates the vertex is the second of the three points.
2.1.3 Measure a line segment.	**2.1.3a** The length of a line segment is the distance between its endpoints. **2.1.3b** The length of a line segment is measured using a specified unit of measure. Units of measure can be formal or informal.
2.1.4 Measure an angle.	**2.1.4a** An angle can be measured by determining the amount of rotation one ray would make about the vertex of the angle to coincide with the other ray. The amount of rotation is measured as a fraction of the rotation needed to rotate a full circle. **2.1.4b** An angle can be measured with reference to a circle whose center is the vertex of the angle by determining the fraction of the circular arc between the intersection points of the rays and the circle. The length of the circular arc is measured as a fraction of the circle's circumference. **2.1.4c** An angle can be measured in units of radians, equaling the arc length spanned by the angle when its vertex coincides with the center of a unit circle.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.1.5 Prove whether two or more line segments are congruent.	**2.1.5a** Two line segments are congruent if and only if one segment can be translated, rotated, or reflected to coincide with the other segment without changing the length of either line segment. **2.1.5b** Two line segments are congruent if and only if they have the equal lengths.
2.1.6 Prove whether two or more angles are congruent.	**2.1.6a** Two angles are congruent if and only if one angle can be translated, rotated, or reflected to coincide with the other angle without changing the measure of either angle. **2.1.6b** Two angles are congruent if and only if they have equal measures.
2.1.7 Construct a congruent copy of a line segment or an angle.	**2.1.7a** A synthetic geometric construction utilizes only a straightedge and a compass to accurately draw or copy a figure. **2.1.7b** A straightedge is a tool for connecting two distinct points with a line segment. **2.1.7c** A compass is a tool for copying distances between pairs of points.
2.1.8 Calculate the distance between two points.	**2.1.8a** The distance between two points in the plane is the length of the line segment connecting the points. **2.1.8b** The distance between two points in the coordinate plane can be determined by applying the Pythagorean theorem to a right triangle whose hypotenuse is a line segment formed by the two points and whose sides are parallel to each axis.
2.1.9 Solve problems involving segment lengths and/or angle measures.	**2.1.9a** Given line segment $\overline{AC}$ and a point, *B*, that lies on the segment between points *A* and *C*, the measure of segment $\overline{AC}$ is the sum of the measures of segments $\overline{AB}$ and $\overline{BC}$. **2.1.9b** Given $\angle AOC$ and ray $\overrightarrow{OB}$ that lies between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, the measure of $\angle AOC$ is equal to the sum of the measures of $\angle AOB$ and $\angle BOC$. **2.1.9c** Two angles are called complementary if the sum of their measures is 90°. Two angles are complementary if they form a right angle when adjacent. **2.1.9d** Two angles are called supplementary if the sum of their measures is 180°. Two angles are supplementary if they form a straight angle when adjacent.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.1.10 Solve problems involving a segment bisector or an angle bisector.	**2.1.10a** The midpoint of a line segment is the point located on the line segment equidistant from the endpoints. **2.1.10b** In the coordinate plane, the *x*- and *y*- coordinates of the midpoint of a line segment are the arithmetic means of the corresponding coordinates of the endpoints. **2.1.10c** A bisector of an angle is a line, ray, or line segment that contains the vertex of the angle and divides the angle into two congruent adjacent angles. **2.1.10d** Points that lie on the angle bisector are equidistant from the sides of the angle.

Content Boundary: When a mathematical statement includes the phrase "if and only if" to join two sentences, it means that the sentences are logically equivalent. That is, both sentences are true or both sentences are false. These sentences are sometimes referred to as "biconditional statements." Students are expected to know that these statements are both true or both false, but it is not necessary for them to know the term "biconditional" for this course.

KEY CONCEPT 2.2: PARALLEL AND PERPENDICULAR LINES

Determining if and how lines intersect to analyze spatial relationships in the real world

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.2.1 Justify the relationship between the slopes of parallel or perpendicular lines in the coordinate plane using transformations.	**2.2.1a** The relationship between the slopes of parallel lines can be justified by comparing their slope triangles using translation. **2.2.1b** The relationship between the slopes of perpendicular lines can be justified by comparing their slope triangles using rotation by 90°.
2.2.2 Solve problems involving two or more parallel lines, rays, or line segments.	**2.2.2a** Two distinct lines, rays, or line segments in the coordinate plane are parallel if and only if they have the same slope or are both vertical. **2.2.2b** A transversal is a line that intersects a set of lines. Two lines, rays, or line segments intersected by a transversal will be parallel if and only if the same-side interior angles formed by the lines and the transversal are supplementary. **2.2.2c** Two lines intersected by a transversal will be parallel if and only if the corresponding angles, alternate interior angles, or alternate exterior angles formed by the lines and the transversal are congruent.
2.2.3 Construct a line, ray, or line segment parallel to another line, ray, or line segment that passes through a point not on the given line, ray, or line segment.	**2.2.3a** Given a line and a point not on the given line, there is exactly one line through the point that will be parallel to the given line. **2.2.3b** Two parallel lines, rays, or line segments in the coordinate plane will have equal slopes and contain no common points.
2.2.4 Solve problems involving the triangle sum theorem.	**2.2.4a** The sum of the interior angles of a triangle in a plane is 180°.
2.2.5 Solve problems involving two or more perpendicular lines, rays, or line segments.	**2.2.5a** A line, ray, or line segment is perpendicular to another line, ray, or line segment if and only if they form right angles at the point where the two figures intersect. **2.2.5b** A line, ray, or line segment is perpendicular to another line, ray, or line segment in the coordinate plane if and only if the two figures intersect and their slopes are opposite reciprocals of each other, or if one is vertical and other is horizontal.
2.2.6 Construct the perpendicular bisector of a line segment.	**2.2.6a** The perpendicular bisector of a line segment intersects the line segment at its midpoint and forms four right angles with the line segment. **2.2.6b** The perpendicular bisector of a line segment is determined by identifying two points in a plane that are equidistant from the endpoints of the line segment and constructing a line, ray, or line segment through those two points. **2.2.6c** Every point that lies on the perpendicular bisector of a line segment is equidistant from the endpoints of the line segment.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.2.7 Construct a line, ray, or line segment perpendicular to another line, ray, or line segment.	**2.2.7a** A horizontal line, ray, or line segment in the coordinate plane is perpendicular to a vertical line, ray, or line segment if they intersect. **2.2.7b** Two perpendicular lines, rays, or line segments in the coordinate plane will intersect and have slopes that are opposite reciprocals of each other, or one will be vertical and the other will be horizontal. **2.2.7c** Applying the perpendicular bisector construction to a point on a line, ray, or line segment is sufficient to construct a line, ray, or line segment perpendicular to the given line, ray, or line segment.

Content Boundary: Some learning objectives in this key concept require students to create both synthetic and analytic arguments for geometric relationships. Pre-AP expects students to use tools and techniques of synthetic geometry to determine, justify, or explain relationships of figures studied in a plane without coordinates and to use tools and techniques of analytic geometry to determine, justify, or explain relationships of figures studied in the coordinate plane. Students are expected to develop proficiency in both realms and to move fluently between them. However, it is not necessary that they use the terms *synthetic* and *analytic*.

Cross Connection: In Pre-AP Algebra 1, students made extensive use of *slope triangles* – right triangles whose legs are parallel to the axes of the coordinate plane – to calculate the slope of a non-vertical and non-horizontal line. In this course, students connect their prior knowledge of slope triangles with their understanding of geometric transformations to gain new insights into the relationships between the slopes of parallel and perpendicular lines.

KEY CONCEPT 2.3: MEASUREMENT IN RIGHT TRIANGLES

Using the relationships between the side lengths and angle measures of right triangles to create new measurements

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.3.1 Prove whether two right triangles are similar using informal similarity transformations.	**2.3.1a** Two right triangles are similar if and only if one triangle can be translated, reflected, and/or rotated so it coincides with the other after dilating one triangle by a scale factor. **2.3.1b** Two right triangles are similar if and only if their corresponding angles have equal measures. **2.3.1c** Two right triangles are similar if and only if their corresponding side lengths are in proportion.
2.3.2 Determine the coordinates of a point on a line segment.	**2.3.2a** The coordinates of a point along a line segment in the coordinate plane that divides the line segment into a given ratio can be determined using similar triangles.
2.3.3 Prove the Pythagorean theorem using similar right triangles.	**2.3.3a** An altitude drawn from the right angle of a right triangle to the hypotenuse creates similar right triangles. **2.3.3b** When an altitude is constructed from the right angle to the hypotenuse of a right triangle, the proportions of the side lengths of the similar right triangles formed can be used to prove the Pythagorean theorem.
2.3.4 Associate the measures of an acute angle, $\angle A$, in a right triangle to ratios of the side lengths.	**2.3.4a** The sine of the measure of $\angle A$ is the ratio of the length of the side opposite the angle and the length of the hypotenuse. **2.3.4b** The cosine of the measure of $\angle A$ is the ratio of the length of the side adjacent to the angle and the length of the hypotenuse. **2.3.4c** The tangent of the measure of $\angle A$ is the ratio of the length of the side opposite the angle and the length of the side adjacent to the angle.
2.3.5 Explain why a trigonometric ratio depends only on an angle measure of a right triangle and not on the side lengths.	**2.3.5a** Trigonometric ratios are functions whose input is an acute angle measure and whose output is a ratio of two side lengths in a right triangle. **2.3.5b** The ratio of the lengths of two sides of a right triangle will equal the ratio of the lengths of the corresponding sides of a similar right triangle. Therefore, the ratios of the sides depend only on the angle measure.
2.3.6 Determine an acute angle measure in a right triangle, given a ratio of its side lengths, using an understanding of inverses.	**2.3.6a** For acute angles in a right triangle, the angle measure and the ratio of the lengths of any two specific sides have a one-to-one correspondence. **2.3.6b** Given a ratio of any two side lengths in a right triangle, it is possible to determine the acute angle measures of the right triangle.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.3.7 Model contextual scenarios using right triangles.	**2.3.7a** Contextual scenarios that involve nonvertical and nonhorizontal segments or the distance between two points that do not lie on a vertical or horizontal line can be modeled by right triangles. **2.3.7b** Trigonometric ratios can be used to solve problems or model scenarios involving angles of elevation or depression.

Content Boundary: Formally defining inverse trigonometric functions is beyond the scope of this course. However, students should understand that because each acute angle in a right triangle is uniquely associated with a specific ratio of side lengths, then a ratio of side lengths can be used to determine a specific acute angle in a right triangle. That is, students should be expected to "go forward" by determining the sine, cosine, or tangent of an acute angle and to "go backward" by determining the acute angle whose sine, cosine, or tangent ratio is given. Students are expected to use a scientific calculator to determine an angle measure, given a trigonometric ratio.

Unit 3: Measurement in Congruent and Similar Figures

Suggested Timing: Approximately 7 weeks

Informal transformations are the way we, as humans, compare two objects to see if they are congruent. We turn, twist, and flip objects to see if one can lay exactly on the other without bending, stretching, or breaking either object. When they match, we say the objects are congruent. If they do not match, but they have the same shape and the same scaled measurements, we say the objects are similar. Transformations in geometry give us language to describe these turns, twists, flips, and scaling precisely and systematically. This unit formalizes the concept of congruence and similarity of planar objects by identifying the essential components of rigid motion and similarity transformations. Students are expected to become proficient with transformations that involve coordinates as well as with transformations that do not involve coordinates. Throughout the course, transformations are presented as functions. This connection further develops students' understanding of functions and connects the statistics and geometry units of the course. It also creates a bridge between Algebra 1 and Algebra 2 since concept of function permeates and links nearly all aspects of high school mathematics. Students develop further insights into congruence and similarity by exploring which transformations affect angle measures and distances between pairs of points and which do not. Students apply their understandings of transformations, congruence, and similarity to solve problems involving polygons and circles.

Throughout Units 2–4, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables, flowcharts, or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.

ENDURING UNDERSTANDINGS

Students will understand that …

- Transformations are functions that can affect the measurements of a geometric figure.
- Congruent figures have equal corresponding angle measures and equal distances between corresponding pairs of points.
- Similar figures have equal corresponding angle measures, and the distances between corresponding pairs of points are proportional.
- The geometry of a circle is completely determined by its radius.

KEY CONCEPTS

- **3.1: Transformations of points in a plane** – Defining transformations to describe the movement of points and shapes
- **3.2: Congruent and similar polygons** – Using transformations to compare figures with the same size or same shape
- **3.3: Measurement of lengths and angles in circles** – Using measurements in circles to make sense of round flat objects in the physical world

KEY CONCEPT 3.1: TRANSFORMATIONS OF POINTS IN A PLANE

Defining transformations to describe the movement of points and shapes

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.1.1 Perform transformations on points in a plane.	**3.1.1a** Transformations describe motions in the plane. Analyzing these transformations indicates if and how these motions affect lengths and angle measures of figures. Congruence and similarity are defined in terms of measurements that are preserved by transformations. **3.1.1b** A transformation is a function whose inputs and outputs are points in the plane. A set of all input points of a transformation is called a preimage; a set of all output points of the preimage is called an image. **3.1.1c** A rigid motion transformation preserves both the distance between pairs of points and the angle measures. A similarity transformation preserves angle measures but not necessarily distances between pairs of points.
3.1.2 Express transformations using function notation.	**3.1.2a** Given a transformation T and two points, A and B, the notation $T(A)=B$ means that the image of point A under transformation T is point B. The transformation is said to map point A to point B. **3.1.2b** Algebra can be used to express how a transformation affects the x- and y-coordinates of points. All transformations can be represented using function notation, but some transformations are difficult to define as algebraic expressions.
3.1.3 Prove that a rigid motion transformation maps an object to a congruent object.	**3.1.3a** A rigid motion transformation is a transformation that preserves distances between pairs of points as well as angle measures. **3.1.3b** A translation is a transformation that maps each point in the plane to an image that is a specified distance in a specified direction from the preimage. **3.1.3c** A reflection is a transformation that maps each point in the plane to its mirror image across a line called the axis of symmetry. **3.1.3d** A rotation is a transformation that maps each point in the plane to an image that is turned by a specified angle about a fixed point called the center of rotation.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.1.4 Solve problems involving rigid motion transformations.	**3.1.4a** Applying one or more translations, rotations, and reflections maps an object to a congruent object. **3.1.4b** Any transformation that preserves distance between points and angle measures can be written as a sequence of translations, reflections, and/or rotations. **3.1.4c** If two figures are congruent, there must exist a sequence of one or more rigid motion transformations that maps one figure to the other.
3.1.5 Prove that a similarity transformation maps an object to a similar object.	**3.1.5a** A similarity transformation is a sequence of a dilation and/or one or more rigid motion transformations. **3.1.5b** A dilation from a fixed point, called the center, with a scale factor k is a transformation that maps each point in the plane to an image whose distance from the center is k times the distance between the center and the preimage, in the same direction as the preimage. **3.1.5c** Dilations of figures do not affect the angle measures of a figure.
3.1.6 Solve problems involving similarity transformations.	**3.1.6a** Dilating the plane by a scale factor k with center $(0, 0)$ will scale each coordinate by k. **3.1.6b** A dilation maps a line not passing through the center of the dilation to a parallel line and maps a line passing through the center of dilation to itself. **3.1.6c** The scale factor of a dilation can be determined by dividing a length from the image by its corresponding length in the preimage. **3.1.6d** The perimeter of the image of a figure is the perimeter of the preimage scaled by the same scale factor as the dilation.

Content Boundary: Students are expected to use algebra to express translations in the coordinate plane, reflections across the x-axis, the y-axis, and the line $y = x$, and rotations about the origin, clockwise or counterclockwise, by angles of 90° and 180°. Students are also expected to identify axes of symmetry and angles of rotation beyond those listed above. However, using algebra to express reflections across lines other than those listed, or rotations about angles other than 90° or 180° is beyond the scope of the course. It is most important that students understand that some transformations are difficult to express using algebra, but that function notation can be used to communicate the relationship between the inputs and outputs of any transformation.

KEY CONCEPT 3.2: CONGRUENT AND SIMILAR POLYGONS

Using transformations to compare figures with the same size or same shape

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.2.1 Prove that two triangles are congruent by comparing their side lengths and angle measures.	**3.2.1a** If the three sides and three angles of a triangle are congruent to the three sides and three angles of another triangle, then the two triangles are congruent. **3.2.1b** If two triangles are congruent, then all six corresponding parts of the triangles are also congruent.
3.2.2 Prove that two triangles are congruent by comparing specific combinations of side lengths and angle measures.	**3.2.2a** If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent (SSS). **3.2.2b** If two sides of a triangle and the interior angle they form are congruent to two sides of another triangle and the interior angle they form, then the triangles are congruent (SAS). **3.2.2c** If two angles and the side adjacent to both angles of a triangle are congruent to two angles and the side adjacent to both angles in another triangle, then the triangles are congruent (ASA).
3.2.3 Prove that two triangles are similar.	**3.2.3a** Two triangles are similar if and only if they have three pairs of congruent angles. **3.2.3b** Two triangles are similar if and only if the lengths of their corresponding sides are in proportion. **3.2.3c** Two triangles are similar if and only if one can be mapped to coincide with the other after applying a similarity transformation.
3.2.4 Prove theorems about parallelograms.	**3.2.4a** Proofs about parallelograms are based on relationships among their sides, angles, and diagonals. **3.2.4b** A line segment between two opposite vertices in a parallelogram forms two congruent triangles that share a common side. **3.2.4c** For a parallelogram in the coordinate plane, the slopes of the sides and diagonals can be used to prove statements about the parallelogram.

Content Boundary: This key concept is traditionally the major focus of high school geometry courses. It is certainly valuable that students prove theorems about congruent and similar triangles and quadrilaterals. Students are expected to use a variety of formats to construct mathematical arguments including but not limited to two-column proofs and paragraph proofs. The format of a student's proof is not as important as their ability to justify or provide a counterexample to a mathematical claim.

KEY CONCEPT 3.3: MEASUREMENT OF LENGTHS AND ANGLES IN CIRCLES

Using measurements in circles to make sense of round flat objects in the physical world

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.3.1 Determine whether a particular point lies on a given circle.	**3.3.1a** A point in the coordinate plane lies on a circle if its coordinates satisfy the equation of a circle. **3.3.1b** All points that lie on a circle are equidistant from the center of the circle.
3.3.2 Translate between the geometric and algebraic representations of a circle.	**3.3.2a** A circle is the set of all points equidistant from a given point. **3.3.2b** In the coordinate plane, the graph of the equation $(x-h)^2+(y-k)^2=r^2$ is the set of all points located r units from the point (h, k). This is a circle with radius r and center (h, k).
3.3.3 Prove that any two circles are similar.	**3.3.3a** Every circle can be expressed as the image of any other circle under a similarity transformation.
3.3.4 Determine the measure of a central angle or the circular arc it intercepts.	**3.3.4a** A central angle is an angle whose vertex is the center of a circle and whose sides are, or contain, two radii of the circle. **3.3.4b** The measure of an arc is defined as the measure of the central angle that intercepts the arc.
3.3.5 Determine the measure of an inscribed angle or the circular arc it intercepts.	**3.3.5a** An inscribed angle is an angle whose vertex lies on a circle and whose sides contain chords of the circle. **3.3.5b** The measure of an inscribed angle is half the measure of the arc it intercepts. Equivalently, the measure of the intercepted arc is twice the measure of the inscribed angle. **3.3.5c** Inscribed angles that intercept the same arc have equal angle measures.
3.3.6 Determine the length of a circular arc.	**3.3.6a** The length of a circular arc depends on the measure of the central angle that intercepts the arc and the radius of the circle. **3.3.6b** The ratio of the length of a circular arc and the circumference of the circle is equal to the ratio of the measure of the central angle that intercepts the arc and the angle measure of a full circle.
3.3.7 Construct a line, ray, or line segment tangent to a circle.	**3.3.7a** A line, ray, or line segment tangent to a circle intersects the circle at exactly one point. **3.3.7b** A line, ray, or line segment tangent to a circle is perpendicular to a radius of the circle at the point of intersection. **3.3.7c** In the coordinate plane, the slope of the line, ray, or line segment tangent to the circle and the slope of the radius that intersects this tangent line, ray, or line segment will be opposite reciprocals, or one will be vertical and the other will be horizontal.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.3.8 Solve a system of equations consisting of a linear equation and the equation of a circle.	**3.3.8a** The intersection of a line and a circle corresponds to an algebraic solution of the system of their corresponding equations. **3.3.8b** An algebraic solution to a system of equations is an ordered pair that makes all equations true simultaneously. The system may have zero, one, or two solutions.

Content Boundary: It is beyond the scope of the course for students to know that a unit circle has a radius of length 1, or to know the coordinates of points on the circle that correspond to special reference angles.

Cross Connection: The length of a circular arc, as defined through Learning Objective 3.3.6, explicitly connects students' prior knowledge of ratios to their current study of geometry. It is more important for students to understand that an arc length is proportional to the circumference of the circle than it is for them to memorize a formula relating arc length and central angle measure.

Unit 4: Measurement in Two and Three Dimensions

Suggested Timing: Approximately 7 weeks

This unit deepens students' understanding of measurement by expanding the concept of measurement to two dimensions through the areas of planar figures and to three dimensions through volumes of solid figures. One reason for studying area is that it often represents quantities that are otherwise difficult to compute. For example, the area under the graph of an object's speed corresponds to its total distance traveled. Therefore, techniques for calculating area can be adapted to find other quantities. Students likely have prior experience with calculating the areas of conventional figures and composites of those figures. Students may also have experience calculating the volumes of conventional solids. The unit introduces students to Cavalieri's principle, which relates the area of a figure to its cross-sectional lengths and the volume of a solid to its cross-sectional areas. The focus of the unit is on justifying area and volume formulas with which students are already familiar and using area and volume to model real-world physical scenarios.

Throughout Units 2-4, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables, flowcharts, or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.

ENDURING UNDERSTANDINGS

Students will understand that …

- The area of a figure depends on its height and its cross-sectional widths.
- The volume of a solid depends on its height and its cross-sectional areas.
- The geometry of a sphere is completely determined by its radius.

KEY CONCEPTS

- **4.1: Area as a two-dimensional measurement –** Connecting one- and two-dimensional measurements to develop an understanding of area as a measurement of flat coverage
- **4.2: Volume as a three-dimensional measurement –** Connecting two- and three-dimensional measurements to develop an understanding of volume as a measurement of space occupied
- **4.3: Measurements of spheres –** Measuring areas and volumes of spheres to make sense of round objects in the physical world

KEY CONCEPT 4.1: AREA AS A TWO-DIMENSIONAL MEASUREMENT

Connecting one- and two-dimensional measurements to develop an understanding of area as a measurement of flat coverage

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
4.1.1 Use Cavalieri's principle to solve problems involving the areas of figures.	**4.1.1a** If two figures have congruent bases and equal heights, and line segments in the interiors of those figures that are parallel to, and equal distances from the base are congruent, then the figures will have equal area.
4.1.2 Determine the area of a sector.	**4.1.2a** The area of a sector depends on the measure of the central angle that forms the sector and the radius of the circle. **4.1.2b** The ratio of the area of a sector and the area of the circle is equal to the ratio of the measure of the central angle that forms the sector and the angle measure of a full circle.
4.1.3 Determine the effect of a similarity transformation on the area of a figure.	**4.1.3a** The area of the image of a figure is the area of the preimage scaled by the square of the scale factor of the dilation.

Content Boundary: Prior to this course, students will have explored the area formulas for planar figures, such as triangles, quadrilaterals, other polygons, and circles. In this course, students deepen their understanding of area by connecting length measures within a figure to its area, and by using area to solve real-world problems.

Cross Connection: The area of a sector, as defined through Learning Objective 4.1.2, explicitly connects students' prior knowledge of ratios to their current study of geometry. As with the length of a circular arc in Unit 3, it is more important that students understand that the area of a sector is proportional to the area inside the circle than it is for them to memorize the related formula.

KEY CONCEPT 4.2: VOLUME AS A THREE-DIMENSIONAL MEASUREMENT

Connecting two- and three-dimensional measurements to develop an understanding of volume as a measurement of space occupied

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.2.1 Justify the volume formula for a right prism.	**4.2.1a** The cross section of a right prism is the polygon formed by the intersection of the solid with a plane parallel to its base. **4.2.1b** The volume of a right prism is equal to the product of the height of the solid and the area of its base.
4.2.2 Justify the volume formula for pyramids.	**4.2.2a** The cross section of a pyramid is the polygon formed by the intersection of the solid with a plane parallel to its base. **4.2.2b** The volume of a pyramid is equal to one-third of the volume of its associated prism. That is, the volume of a pyramid is one-third of the product of the height of the solid and the area of its base.
4.2.3 Justify the volume formula for a right cylinder.	**4.2.3a** The cross section of a right cylinder is the circle formed by the intersection of the solid with a plane parallel to its base. **4.2.3b** The volume of a right cylinder is equal to the product of the height of the solid and the area of its base.
4.2.4 Justify the volume formula for a cone.	**4.2.4a** The cross section of a cone is the circle formed by the intersection of the solid with a plane parallel to its base. **4.2.4b** The volume of a cone is equal to one-third of the volume of its associated cylinder. That is, the volume of a cone is one-third of the product of the height of the solid and the area of its base.
4.2.5 Use Cavalieri's principle to solve problems involving volumes of solids.	**4.2.5a** If two solid figures have congruent bases and equal heights, and cross sections that are parallel to and equal distances from each base are congruent, then the solids have equal volume.
4.2.6 Solve contextual problems involving volume of solid figures.	**4.2.6a** Physical objects in many real-world scenarios can be modeled by solid geometric figures such as prisms, pyramids, cylinders, and cones.

Cross Connection: The concept of Cavalieri's principle, which students use to solve problems involving volumes of solids, connects the area of a cross section of a solid to the volume of that solid. Understanding the relationship between the area of a cross section and the volume of a solid will help students who progress to AP Calculus make sense of why finding volumes of solids is an application of integration.

KEY CONCEPT 4.3: MEASUREMENTS OF SPHERES

Measuring areas and volumes of spheres to make sense of round objects in the physical world

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.3.1 Define spheres in terms of distance.	**4.3.1a** A sphere is an object in three-dimensional space that is the set of all points equidistant from a given point, called its center.
4.3.2 Justify the surface area formula for a sphere.	**4.3.2a** The surface area of a sphere is given by the formula $SA = 4\pi r^2$, where r represents the length of the radius of the sphere.
4.3.3 Justify the volume formula for a solid sphere.	**4.3.3a** The volume of a solid sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r represents the length of the radius of the sphere.
4.3.4 Solve contextual problems using spheres.	**4.3.4a** Round physical objects in real-world scenarios can be modeled by spheres.

Content Boundary: It is likely that students have some familiarity with the surface area and volume formulas for a sphere. The focus of this key concept is for students to develop an informal understanding of the derivation of the surface area and volume formulas for spheres and to use spheres to model physical scenarios.

Pre-AP Geometry with Statistics Model Lessons

Model lessons in Pre-AP Geometry with Statistics are developed in collaboration with geometry and statistics educators across the country and are rooted in the course framework, shared principles, and areas of focus. Model lessons are carefully designed to illustrate on-grade-level instruction. Pre-AP strongly encourages teachers to internalize the lessons and then offer the supports, extensions, and adaptations necessary to help all students achieve the lesson goals.

The purpose of these model lessons is twofold:

- **Robust instructional support for teachers:** Pre-AP Geometry with Statistics model lessons are comprehensive lesson plans that, along with accompanying student resources, embody the Pre-AP approach to teaching and learning. Model lessons provide clear and substantial instructional guidance to support teachers as they engage students in the shared principles and areas of focus.
- **Key instructional strategies:** Commentary and analysis embedded in each lesson highlight not just what students and teachers do in the lesson, but also how and why they do it. This educative approach provides a way for teachers to gain unique insight into key instructional moves that are powerfully aligned with the Pre-AP approach to teaching and learning. In this way, each model lesson works to support teachers in the moment of use with students in their classroom.

Teachers have the option to use any or all model lessons alongside their own locally developed instructional resources. Model lessons target content areas that tend to be challenging for teachers and students. While the lessons are distributed throughout all four units, they are concentrated more heavily in the beginning of the course to support teachers and students in establishing a strong foundation in the Pre-AP approach to teaching and learning.

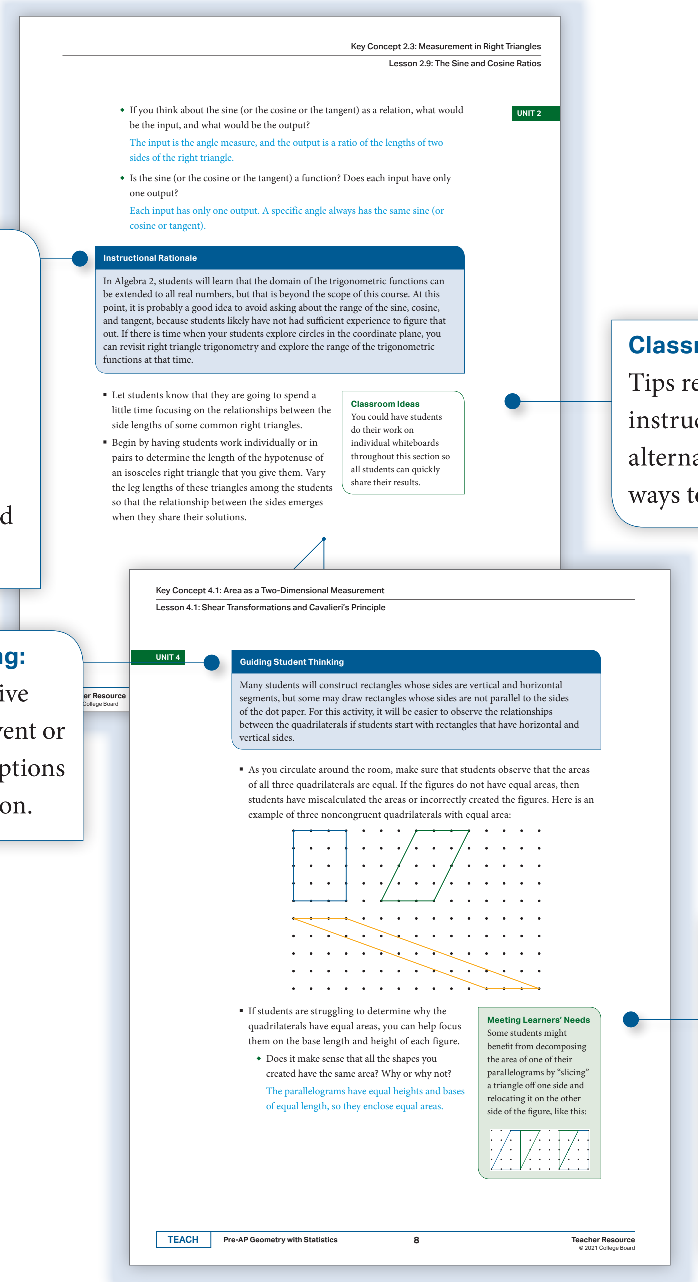
SUPPORT FEATURES IN MODEL LESSONS

The following support features recur throughout the Pre-AP Geometry with Statistics lessons, to promote teacher understanding of the lesson design and provide direct-to-teacher strategies for adapting lessons to meet their students' needs:

- Instructional Rationale
- Guiding Student Thinking
- Meeting Learners' Needs
- Classroom Ideas

Instructional Rationale: Insight into the strategic design and purpose of the instructional choices, flow, and scaffolding within the model lesson. Rationales often describe how a concept is continued later in the lesson or unit.

Guiding Student Thinking: Ways to facilitate productive student thinking and prevent or address student misconceptions in critical areas of the lesson.


Key Concept 2.3: Measurement in Right Triangles

Lesson 2.9: The Sine and Cosine Ratios

UNIT 2

- If you think about the sine (or the cosine or the tangent) as a relation, what would be the input, and what would be the output?

  The input is the angle measure, and the output is a ratio of the lengths of two sides of the right triangle.

- Is the sine (or the cosine or the tangent) a function? Does each input have only one output?

  Each input has only one output. A specific angle always has the same sine (or cosine or tangent).

Instructional Rationale

In Algebra 2, students will learn that the domain of the trigonometric functions can be extended to all real numbers, but that is beyond the scope of this course. At this point, it is probably a good idea to avoid asking about the range of the sine, cosine, and tangent, because students likely have not had sufficient experience to figure that out. If there is time when your students explore circles in the coordinate plane, you can revisit right triangle trigonometry and explore the range of the trigonometric functions at that time.

- Let students know that they are going to spend a little time focusing on the relationships between the side lengths of some common right triangles.
- Begin by having students work individually or in pairs to determine the length of the hypotenuse of an isosceles right triangle that you give them. Vary the leg lengths of these triangles among the students so that the relationship between the sides emerges when they share their solutions.

Classroom Ideas

You could have students do their work on individual whiteboards throughout this section so all students can quickly share their results.

Key Concept 4.1: Area as a Two-Dimensional Measurement

Lesson 4.1: Shear Transformations and Cavalieri's Principle

UNIT 4

Guiding Student Thinking

Many students will construct rectangles whose sides are vertical and horizontal segments, but some may draw rectangles whose sides are not parallel to the sides of the dot paper. For this activity, it will be easier to observe the relationships between the quadrilaterals if students start with rectangles that have horizontal and vertical sides.

- As you circulate around the room, make sure that students observe that the areas of all three quadrilaterals are equal. If the figures do not have equal areas, then students have miscalculated the areas or incorrectly created the figures. Here is an example of three noncongruent quadrilaterals with equal area:

- If students are struggling to determine why the quadrilaterals have equal areas, you can help focus them on the base length and height of each figure.
  - Does it make sense that all the shapes you created have the same area? Why or why not?

    The parallelograms have equal heights and bases of equal length, so they enclose equal areas.

Meeting Learners' Needs

Some students might benefit from decomposing the area of one of their parallelograms by "slicing" a triangle off one side and relocating it on the other side of the figure, like this:

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Classroom Ideas: Tips related to the logistics of the instruction, such as suggestions for alternative presentation methods or ways to alleviate pacing concerns.

Meeting Learners' Needs: Optional differentiation strategies to address diverse learning needs, such as ideas for just-in-time skill building during a lesson or ways to break a task into smaller tasks, if needed, to make it more accessible.

Pre-AP Geometry with Statistics Assessments for Learning

Pre-AP Geometry with Statistics assessments function as a component of the teaching and learning cycle. Progress is not measured by performance on any single assessment. Rather, Pre-AP Geometry with Statistics offers a place to practice, to grow, and to recognize that learning takes time. The assessments are updated and refreshed periodically.

LEARNING CHECKPOINTS

Based on the Pre-AP Geometry with Statistics Course Framework, the learning checkpoints require students to examine data, models, diagrams, and short texts—set in authentic contexts—in order to respond to a targeted set of questions that measure students' application of the key concepts and skills from the unit. All eight learning checkpoints are automatically scored, with results provided through feedback reports that contain explanations of all questions and answers as well as individual and class views for educators. Teachers also have access to assessment summaries on Pre-AP Classroom, which provide more insight into the question sets and targeted learning objectives for each assessment event.

The following tables provide a synopsis of key elements of the Pre-AP Geometry with Statistics learning checkpoints.

Format	Two learning checkpoints per unit Digitally administered with automated scoring and reporting Questions target both concepts and skills from the course framework
Time Allocated	Designed for one 45-minute class period per assessment
Number of Questions	10–12 questions per assessment ▪ 7–9 four-option multiple choice ▪ 3–5 technology-enhanced questions

Domains Assessed	
Learning Objectives	Learning objectives within each key concept from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus are assessed regularly across all eight learning checkpoints: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

Question Styles	Question sets consist of two to three questions that focus on a single stimulus or group of related stimuli, such as diagrams, graphs, or tables. Questions embed mathematical concepts in real-world contexts. *Please see page 71 for a sample question set that illustrates the types of questions included in Pre-AP learning checkpoints and the Pre-AP final exam.*

PERFORMANCE TASKS

Each unit includes one performance-based assessment designed to evaluate the depth of student understanding of key concepts and skills that are not easily assessed in a multiple-choice format.

These tasks, developed for high school students across a broad range of readiness levels, are accessible while still providing sufficient challenge and the opportunity to practice the analytical skills that will be required in AP mathematics courses and for college and career readiness. Teachers participating in the official Pre-AP Program will receive access to online learning modules to support them in evaluating student work for each performance task.

Format	One performance task per unit Administered in print Educator scored using scoring guidelines
Time Allocated	Approximately 45 minutes or as indicated
Number of Questions	An open-response task with multiple parts

Domains Assessed	
Key Concepts	Key concepts and prioritized learning objectives from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

PRACTICE PERFORMANCE TASKS

One or more practice performance tasks in each unit provide students with the opportunity to practice applying skills and knowledge in a context similar to a performance task, but in a more scaffolded environment. These tasks include strategies for adapting instruction based on student performance and ideas for modifying or extending tasks based on students’ needs.

SAMPLE PERFORMANCE TASK AND SCORING GUIDELINES

The following task and set of scoring guidelines are representative of what students and educators will encounter on the performance tasks. (The example below is a practice performance task in Unit 1.)

PRACTICE PERFORMANCE TASK

Staffing the Grocery Store

OVERVIEW

DESCRIPTION

In this practice performance task, students describe a data distribution, analyze how two different representations of the same distribution can reveal different aspects of it, and conclude that the empirical rule is not an if and only if statement.

CONTENT FOCUS

This task is designed to assess students' understanding of different measures of center and how measures of center are related to the shape of a distribution. This task also reinforces students' understanding of the empirical rule. It is intended to be used at the completion of Key Concept 1.1 in Unit 1. In the task, students construct a histogram, describe a distribution, and compare two distributions. This task also requires students to investigate whether the empirical rule by itself can be used to determine if a distribution is approximately normally distributed.

AREAS OF FOCUS

- Greater Authenticity of Applications and Modeling
- Engagement in Mathematical Argumentation
- Connections Among Multiple Representations

SUGGESTED TIMING

~45 minutes

HANDOUT

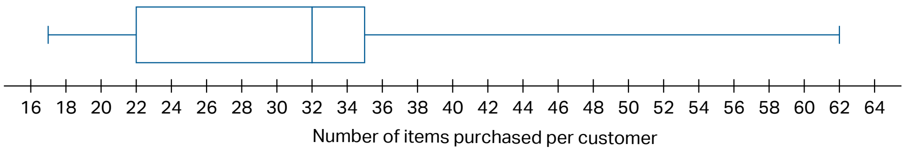
Unit 1 Practice Performance Task: Staffing the Grocery Store

MATERIALS

- scientific calculator or graphing utility
- graph paper (optional)

Practice Performance Task

To determine the number of cashiers needed when a grocery store opens each morning, the manager of the store wanted to estimate the typical number of items each customer purchased. The manager took all the receipts from the first 30 minutes the store was open on a particular day and recorded the number of items purchased by each customer. The manager constructed the following boxplot from the information on the receipts.



(a) Describe the distribution of the number of items purchased per customer. Be sure to indicate the center, shape, and variability of the distribution.

(b) The manager suspected that the distribution of the number of items purchased per customer was approximately normally distributed. Based on your description of the distribution, do you agree with the manager? Explain your reasoning.

(c) The manager lets you review the receipts used to make the boxplot. The actual numbers of items purchased by the customers in the first 30 minutes the store was open were: 17, 18, 20, 21, 21, 22, 24, 25, 26, 27, 31, 33, 33, 34, 34, 35, 35, 37, 42, 42, 46, and 62. Calculate the following statistics:

Mean:

Median:

Standard deviation:

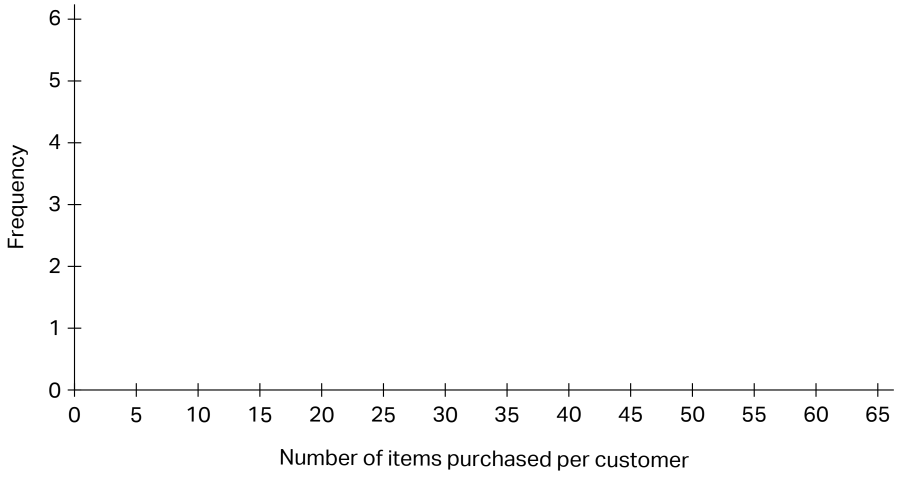
Proportion of customers that purchased items within 1 standard deviation of the mean:

Proportion of customers that purchased items within 2 standard deviations of the mean:

Proportion of customers that purchased items within 3 standard deviations of the mean:

Explain how the statistics you calculated support or contradict your answer from part (b).

(d) On the grid provided, construct a histogram for the number of items purchased per customer. Use bin widths of 5, starting at 15. Based on your histogram and your answers to parts (a) through (c), what can be said about a distribution that follows the empirical rule? Explain your conclusion.



SCORING GUIDELINES

There are 12 possible points for this performance task.

Student Stimulus and Part (a)

(a) Describe the distribution of the number of items purchased per customer. Be sure to indicate the center, shape, and variability of the distribution.

Sample Solutions	Points Possible
Description of center: The median number of items purchased is 32. Since the distribution appears to be skewed right, we would expect the mean to be greater than the median. Description of shape: The distribution of the number of items purchased per customer appears to be skewed right because the right side of the boxplot is longer than the left side of the boxplot. Description of variability: The interquartile range is the difference between the upper and lower quartile values (also called quartile 3 and quartile 1), which is $35 - 22 = 13$, and the range is the difference between the maximum and minimum values, which is $62 - 17 = 45$.	**3 points maximum** 1 point for identifying the median OR for explaining that the shape suggests the mean is greater than the median 1 point for describing the shape of the distribution 1 point for identifying the interquartile range OR for identifying the range as a measure of spread *Scoring note*: A student may simply state the median to receive the point for the description of center. It is not necessary to state the relationship between the mean and the median. If students use an incorrect definition of the range, such as "the range is from 17 to 62," the student should not receive the point for describing the variability.

Targeted Feedback for Student Responses
If several students do not describe all three components of the distribution in part (a), it could mean students need help describing a distribution in terms of its center, shape, and/or variability of a distribution presented as a boxplot.

TEACHER NOTES AND REFLECTIONS

Student Stimulus and Part (b)

(b) The manager suspected that the distribution of the number of items purchased per customer was approximately normally distributed. Based on your description of the distribution, do you agree with the manager? Explain your reasoning.

Sample Solutions	Points Possible
I do not agree with the manager that the distribution is approximately normal. A normal distribution is symmetric with a mean that is equal to the median. Since this distribution appears to be skewed right, I would expect the mean to be greater than the median.	**3 points maximum** 1 point for describing a normal distribution as symmetric 1 point for correctly describing the relationship between the mean and median in a skewed or symmetric distribution 1 point for concluding that the distribution does not appear to be approximately normal based on the skewness and the fact that the mean appears to be greater than the median *Scoring note*: Students should receive full credit if they have already stated the relationship between the mean and median in part (a).

Targeted Feedback for Student Responses
If several students make mistakes in part (b), it could mean students need help connecting the symmetry or skew of a distribution to an understanding of the normal distribution.

TEACHER NOTES AND REFLECTIONS

Student Stimulus and Part (c)

(c) The manager lets you review the receipts used to make the boxplot. The actual numbers of items purchased by the customers in the first 30 minutes the store was open were: 17, 18, 20, 21, 21, 22, 24, 25, 26, 27, 31, 33, 33, 34, 34, 35, 35, 37, 42, 42, 46, and 62. Calculate the following statistics:

Mean:

Median:

Standard deviation:

Proportion of customers that purchased items within 1 standard deviation of the mean:

Proportion of customers that purchased items within 2 standard deviations of the mean:

Proportion of customers that purchased items within 3 standard deviations of the mean:

Explain how the statistics you calculated support or contradict your answer from part (b).

<table>
<tr><th>Sample Solutions</th><th>Points Possible</th></tr>
<tr><td>
The distribution of the numbers of items purchased by customers has these statistics:

mean ≈ 31.136 items.
median = 32 items
standard deviation ≈ 10.476 items

The intervals around the standard deviation are:

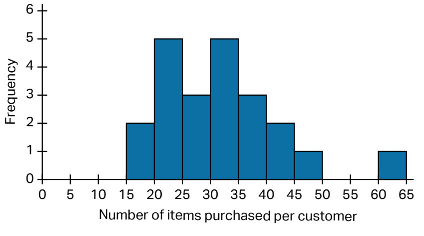
$\mu \pm \sigma$: from 20.660 to 41.612 items
$\mu \pm 2\sigma$: from 10.184 to 52.088 items
$\mu \pm 3\sigma$: from −0.292 to 62.564 items

There are 15 data points within one standard deviation of the mean, 21 data points within two standard deviations of the mean, and 22 data points with three standard deviations of the mean. The corresponding proportions are approximately 0.6818, 0.9545, and 1.000.
These statistics contradict my conclusion from part (b). The mean and the median are approximately equal and the proportions of data within 1, 2, and 3 standard deviations are approximately 68%, 95%, and 99.7%, respectively, which is what we would predict in a normal distribution.
</td><td>
3 points maximum
1 point for determining all three statistics: the mean, median, and standard deviation
1 point for determining all three of the intervals and calculating the proportions within those intervals
1 point for concluding that the mean and median are actually close to each other and that the proportions are what we would expect from a distribution that was approximately normal
Scoring note: If students miscalculate the mean or standard deviation, but use their incorrect mean or standard deviation in the correct manner to determine intervals and the proportion of data in those intervals, the maximum score a student can receive in part (c) is 2.
</td></tr>
<tr><th colspan="2">Targeted Feedback for Student Responses</th></tr>
<tr><td colspan="2">If several students make mistakes determining the summary statistics, it could mean students need practice calculating the mean, median, or standard deviation. If several students cannot conclude that the proportions of data in the interval are consistent with the empirical rule, then they may need additional practice with the normal distribution.</td></tr>
</table>

TEACHER NOTES AND REFLECTIONS

Student Stimulus and Part (d)

(d) On the grid provided, construct a histogram for the number of items purchased per customer. Use bin widths of 5, starting at 15. Based on your histogram and your answers to parts (a) through (c), what can be said about a distribution that follows the empirical rule? Explain your conclusion.

Sample Solutions	Points Possible
A histogram for the number of items purchased per customer is shown below.  Although the distribution is consistent with the empirical rule, the distribution is not approximately normal. It appears to be skewed right, with an extreme value on the right side of the histogram. We can conclude that some distributions have intervals around the mean that correspond to one, two, and three standard deviations that include 68%, 95%, and 99.7% of the data, but are not normally distributed.	**3 points maximum** 1 point for constructing the histogram 1 point for stating the histogram is skewed right 1 point for concluding that a distribution need not be approximately normal even if approximately 68% of the data falls within one standard deviation of the mean, approximately 95% of the data falls within two standard deviations of the mean, and approximately 99.7% of the data falls within 3 standard deviations of the mean *Scoring note*: If students do not mention an extreme value but do describe the distribution as skewed right, the student can still score 3 points in part (d).

Continues on next page.

Targeted Feedback for Student Responses
If several students make mistakes in part (d), they may not understand that having proportions that match the empirical rule proportions is not sufficient for determining whether or not the data is normally distributed. Students could benefit from closer comparisons of normal distributions and other distributions.

TEACHER NOTES AND REFLECTIONS

Points Received	Appropriate Letter Grade (If Graded)	How Students Should Interpret Their Score
11 or 12 points	A	"I know all of this data analysis really well."
8 to 10 points	B	"I know all of this data analysis well, but I made a few mistakes."
5 to 7 points	C	"I know some of this data analysis well, but not all of it."
2 to 4 points	D	"I only know a little bit of this data analysis."
0 or 1 points	F	"I don't know much of this data analysis at all."

FINAL EXAM

Pre-AP Geometry with Statistics includes a final exam featuring multiple-choice and technology-enhanced questions as well as an open-response question. The final exam is a summative assessment designed to measure students' success in learning and applying the knowledge and skills articulated in the Pre-AP Geometry with Statistics Course Framework. The final exam's development follows best practices such as multiple levels of review by educators and experts in the field for content accuracy, fairness, and sensitivity. The questions on the final exam have been pretested, and the resulting data are collected and analyzed to ensure that the final exam is fair and represents an appropriate range of the knowledge and skills of the course.

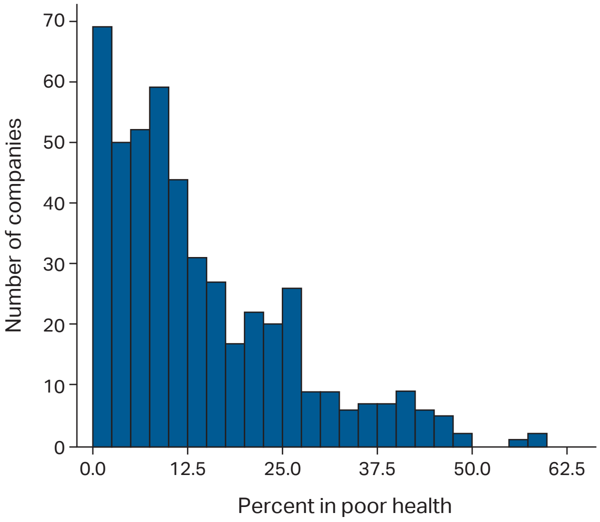
The final exam is designed to be delivered on a secure digital platform in a classroom setting. Educators have the option of administering the final exam in a single extended session or two shorter consecutive sessions to accommodate a range of final exam schedules.

Multiple-choice and technology-enhanced questions are delivered digitally and scored automatically with detailed score reports available to educators. This portion of the final exam is designed to build on the question styles and formats of the learning checkpoints; thus, in addition to their formative purpose, the learning checkpoints provide practice and familiarity with the final exam. The open-response question, modeled after the performance tasks, is delivered as part of the digital final exam but is designed to be scored separately by educators using scoring guidelines that are designed and vetted with the question.

SAMPLE ASSESSMENT QUESTIONS

The following questions are representative of what students and educators will encounter on the learning checkpoints and final exam.

1.



A historian was interested in researching the health of Virginia soldiers near the start of the American Civil War. The data collected for each company, or small group, of soldiers were the percentage of soldiers in each company that were in poor health. A histogram of the data is shown in the figure.

Which of the following statements is most consistent with the distribution of soldiers in poor health?

(A) The total number of companies that reported a percent of soldiers in poor health that was above the mean is equal to the total number of companies that reported a percent of soldiers in poor health below the mean.

(B) The mean and median values for percent of soldiers in poor health are similar.

(C) The variance and the interquartile range of values for percent of soldiers in poor health are similar.

(D) The median value for the percent of soldiers in poor health is less than the mean value.

Assessment Focus

In question 1, students must analyze the distribution of data to determine the relationship between the mean and median based on the skew in the distribution.

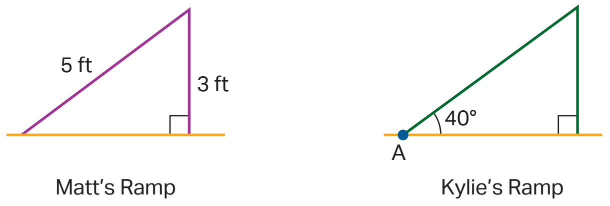
Correct Answer: D

Learning Objective:

1.1.5 Analyze data distributions with respect to their variability.

Area of Focus: Greater Authenticity of Applications and Modeling

2.



Two friends, Matt and Kylie, use different skateboard ramps at their local skate park, as represented in the diagram. They want to know which ramp is steeper. Matt says his ramp has a vertical support (shown) that is 3-feet tall and a ramp length of 5 feet. Kylie measured the angle that the bottom of her ramp made with the ground (angle A) and found it to be 40°.

Based on the known measurements, which ramp is steeper?

(A) Matt's ramp has a slope of $\frac{3}{5}$, which is greater than Kylie's ramp slope of tan(40°).

(B) Matt's ramp has a slope of $\frac{3}{4}$, which is greater than Kylie's ramp slope of sin(40°).

(C) Kylie's ramp has a slope of cos(40°), which is greater than Matt's ramp slope of $\frac{3}{5}$.

(D) Kylie's ramp has a slope of tan(40°), which is greater than Matt's ramp slope of $\frac{3}{4}$.

Assessment Focus

Question 2 requires students to apply the Pythagorean theorem and use the tangent of an angle to determine and compare the slopes of two different segments. Students use these slopes as evidence to support a claim.

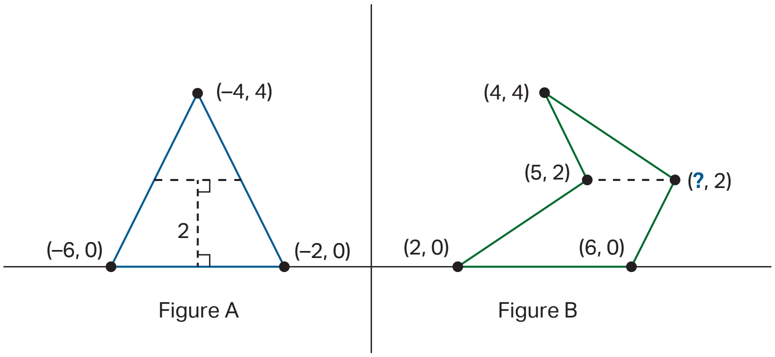
Correct Answer: D

Learning Objective:

2.3.4 Associate the measures of an acute angle, $\angle A$, in a right triangle to ratios of the side lengths.

Area of Focus: Engagement in Mathematical Argumentation

3. The following two figures have the same area.



In the diagram of Figure B, one x-coordinate is represented with a question mark (?). Sahir claims that this missing x-coordinate is 7. Which of the following reasons best supports his claim?

(A) The horizontal distance between points on the sides of Figure B is 2, therefore the missing x-coordinate is 2 more than 5.

(B) The horizontal distances at the same height between two points of Figure A and two points of Figure B are equal.

(C) The slope of the line segment whose endpoints are (6, 0) and (7, 2) is 2.

(D) If points on Figures A and B have the same y-coordinate, then the points will have the same horizontal distance from the y-axis.

Assessment Focus

Question 3 requires students to apply Cavalieri's principle to support a claim about a missing coordinate in a shape.

Correct Answer: B

Learning Objective:

4.1.1 Use Cavalieri's principle to solve problems involving the areas of figures.

Area of Focus: Engagement in Mathematical Argumentation

Pre-AP Geometry with Statistics Course Designation

Schools can earn an official Pre-AP Geometry with Statistics course designation by meeting the program commitments summarized below. Pre-AP Course Audit Administrators and teachers will complete a Pre-AP Course Audit process to attest to these commitments. All schools offering courses that have received a Pre-AP Course Designation will be listed in the Pre-AP Course Ledger, in a process similar to that used for listing authorized AP courses.

PROGRAM COMMITMENTS

- Teachers have read the most recent *Pre-AP Geometry with Statistics Course Guide*.
- The school ensures that Pre-AP frameworks and assessments serve as the foundation for all sections of the course at the school. This means that the school must not establish any barriers (e.g., test scores, grades in prior coursework, teacher or counselor recommendation) to student access and participation in the Pre-AP Geometry with Statistics coursework.
- Teachers administer at least one of two learning checkpoints per unit on Pre-AP Classroom and one performance task per unit.
- Teachers complete the foundational professional learning (Online Foundational Modules or Pre-AP Summer Institute) and at least one online performance task scoring module. The current Pre-AP coordinator completes the Pre-AP Coordinator Online Module.
- Teachers align instruction to the Pre-AP Geometry with Statistics Course Framework and ensure their course meets the curricular commitments summarized below.
- The school ensures that the resource commitments summarized below are met.

CURRICULAR COMMITMENTS

- The course provides opportunities for students to develop understanding of the Pre-AP Geometry with Statistics key concepts and skills articulated in the course framework through the four units of study.
- The course provides opportunities for students to engage in the Pre-AP shared instructional principles.
  - close observation and analysis
  - evidence-based writing
  - higher-order questioning
  - academic conversation

- The course provides opportunities for students to engage in the three Pre-AP mathematics areas of focus. The areas of focus are:
  - greater authenticity of applications and modeling
  - engagement in mathematical argumentation
  - connections among multiple representations
- The instructional plan for the course includes opportunities for students to continue to practice and develop disciplinary skills.
- The instructional plan reflects time and instructional methods for engaging students in reflection and feedback based on their progress.
- The instructional plan reflects making responsive adjustments to instruction based on student performance.

RESOURCE REQUIREMENTS

- The school ensures that participating teachers and students are provided computer and internet access.
- Teachers should have consistent access to a video projector for sharing web-based instructional content and short web videos.

Accessing the Digital Materials

Pre-AP Classroom is the online application through which teachers and students can access Pre-AP instructional resources and assessments. The digital platform is similar to AP Classroom, the online system used for AP courses.

Pre-AP coordinators receive access to Pre-AP Classroom via an access code delivered after orders are processed. Teachers receive access after the Pre-AP Course Audit process has been completed.

Once teachers have created course sections, student can enroll in them via access code. When both teachers and students have access, teachers can share instructional resources with students, assign and score assessments, and complete online learning modules; students can view resources shared by the teacher, take assessments, and receive feedback reports to understand progress and growth.

preap.org/Geometry-CG

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