About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 6,000 of the world’s leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success—including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

For further information, visit www.collegeboard.org

The information included in this guide is still subject to change, as Pre-AP courses are in the refinement and editing stage and still incorporating educator feedback. Pre-AP course materials, including the course framework, lessons, and unit assessments will be finalized in spring 2018. Pre-AP courses will launch in fall 2018.

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Dear Educator:

Thank you for taking the time to review the sample instructional materials for the College Board’s new Pre-AP Program.

**Pre-AP Rationale**

Less than 50% of U.S. high school students are ready for college. Over 300,000 high school students demonstrate AP potential based on their PSAT score but do not take an AP exam. Teachers have told us that they want the College Board’s assistance in helping define what a Pre-AP curriculum should look like. Given these statistics and feedback, the College Board developed the Pre-AP Program.

While the AP Program has helped prepare millions of students for college, data and educator feedback show that we need to reach more students, earlier, because all students deserve access to a challenging curriculum. By offering Pre-AP courses to all ninth-graders, with more grades to come, we hope to provide a new, consistent standard of high-quality instructional resources with the focus on supporting all students so that more of them are ready for college and, when appropriate, able to access and complete college-level work before leaving high school.

Launching in fall 2018, Pre-AP will begin with five ninth-grade courses in World History and Geography, Algebra I, Biology, English, and Arts.

**Goals**

- Significantly increase the number of students who are able to access and complete college-level work before leaving high school
- Improve the college readiness of all students

**Teacher Developed**

We developed the Pre-AP Program in collaboration with educators and teachers like you. Teacher feedback helped us design a program that supplies effective resources and yet gives teachers the freedom and flexibility to teach the way they’ve always wanted to teach.

**What We Provide**

- **Instructional Resources**: Course frameworks, high-quality texts, and source materials paired with effective teaching strategies, model lessons, and shared routines
- **Assessments**: Digital unit assessments and performance-based tasks accompanied by scoring rubrics
- **Student practice**: Resources and tools to help students master content
- **Professional learning**: Training and teacher supports

We hope you find the sample instructional materials useful. As the Pre-AP Program develops, we anticipate that feedback from our school and educator partners will help us strengthen the program to better meet our mutual goal of preparing as many students as possible for success in college. Thank you!

Respectfully,

*The Pre-AP Team*
Getting to Know the Pre-AP Algebra 1 Course

Overview

Pre-AP Algebra 1 focuses deeply on the concepts and skills that have maximum value for college and career. Therefore, a major focus of this course is on mastery of linear relationships. Linear equations and functions represent the most useful and versatile tools in the mathematician’s toolbox, and they are the basic building blocks of many advanced topics in math. The course content has been streamlined to allow time and space to truly master these concepts and skills.

This course creates a platform for all students to build their mathematical muscle and confidence by emphasizing these essential practices for deep mathematical learning:

- Building conceptual understanding
- Building procedural fluency
- Using and making mathematical models
- Crafting mathematical arguments

Pre-AP Algebra 1 asks students to actively construct mathematical knowledge alongside their peers so that students linger productively on problems, persevering through challenges and making important connections among concepts. Students will engage in authentic and useful applications of math as they engage in solving real-world problems. Finally, students will gain the tools for making, testing, refuting, and supporting mathematical arguments, since crafting a mathematical argument lies at the heart of the discipline.

Instructional Shifts for Pre-AP Algebra 1

Pre-AP Algebra 1 instructional resources focus on the following key instructional shifts:

- **Linear Equations and Linear Functions**
  Emphasis on linear equations and linear functions: Students develop deep and robust understanding of linear relationships in procedural, conceptual, and applied settings.

- **Authentic Applications**
  Focus on authentic applications: Students employ mathematics to model and explain authentic scenarios.

- **Mathematical Arguments**
  Concentration on creating mathematical arguments: Students use evidence to craft mathematical conjectures and prove or disprove them.
Shared Instructional Principles

All Pre-AP courses share a common set of classroom routines and approaches that give students many opportunities to practice and strengthen their skills while building their confidence in the classroom.

Close Observation and Analysis

Pre-AP courses require careful examination of one object or problem before requiring students to grapple with multiple. Students engage in deep observation to build, refine, or confirm their knowledge, thus developing a foundational skill that supports analysis and learning in each discipline. As students encounter texts, visual art, graphs, maps, problems, and other source materials, they learn first to engage in deep close observation before being asked to explain and then apply or evaluate.

Evidence-Based Writing

Pre-AP courses provide a scaffolded approach to writing that begins with a focus on the sentence before progressing to more extended-level writing. All courses will provide tools and supports (sentence frames, outlines, and graphic organizers) to support writing skills. In Algebra 1, students will receive ample opportunities in crafting sentence-level written justifications to support or critique mathematical solutions.

Higher-Order Questioning

When examining data, problems, and other sources of evidence, students are guided to grapple with questions that spark curiosity, cultivate wonder, and promote productive lingering. Pre-AP lessons provide teachers with questions that motivate thought and support students to build evidence-based claims and to solve problems from multiple angles.

Academic Conversations

In Pre-AP classrooms, students have frequent opportunities for active, thoughtful participation in collaborative conversations about significant themes, topics, and texts. Through these discussions, students practice the skills of academic conversation that they will need to employ in college and career settings. Students regularly compare, critique, debate, and build upon others’ ideas and arguments to advance their learning.
Course Framework

The Pre-AP Algebra 1 course framework is intended to provide a clear and focused description of what students should know and be able to do as a result of this course.

Based on the Understanding by Design (Wiggins & McTighe, 1998) model, the framework also serves as the blueprint for the instructional units and assessments that are part of the Pre-AP course and is structured in two parts:

Big Ideas and Enduring Understandings

Big Ideas and Enduring Understandings cut across all units of the course. The Big Ideas map out the core principles, theories, and processes of algebra that offer students a broad way of thinking about the discipline. The Enduring Understandings represent the long-term takeaways that students should develop as a result of focused study of the key concepts in the course. By design, Pre-AP Algebra 1 is based on a small, focused set of Big Ideas and Enduring Understandings that can rest on a single page. This instructional design supports deeper learning of concepts and skills and allows students to understand the connections across major principles, processes, and systems in Algebra 1.

Unit Outlines

Unit Outlines articulate the key concepts and learning objectives for each of the four major units of this course. These unit outlines also include general pacing recommendations and mappings to Pre-AP instructional resources to support teacher planning.

The full course framework will be released in spring 2018, but the following section offers a preview.
Big Ideas and Enduring Understandings

The Structure of Real Numbers

**Enduring Understanding 1.A:**
Algebraic manipulations can be used to transform algebraic expressions, equations, and inequalities into equivalent expressions, equations, and inequalities to reveal various aspects of the relationships present.

**Enduring Understanding 1.B:**
A variable is a symbol that can be used to represent a quantity or an expression.

**Enduring Understanding 1.C:**
The set of real numbers consists of all possible decimal numbers and can be divided into rational numbers, those with terminating or repeating decimal representations, and irrational numbers, those with nonterminating and nonrepeating decimal representations.

**Enduring Understanding 1.D:**
Exponents are a useful notation whose properties are derived from the properties of multiplication and division.

Linear Functions

**Enduring Understanding 3.A:**
Linear relationships have a constant rate of change.

**Enduring Understanding 3.B:**
A linear equation in two variables can be represented in algebraic, graphical, and tabular forms.

**Enduring Understanding 3.C.:**
Linear functions are useful for modeling linear contextual situations and for making sense of some nonlinear situations.

**Enduring Understanding 3.D:**
Linear models can be developed from pairs of related data and applied to make predictions in context.

Quadratic Functions

**Enduring Understanding 4.A:**
A quadratic function is formed by the product of two linear expressions.

**Enduring Understanding 4.B:**
The graph of a quadratic function is symmetric about the vertical line containing its maximum or minimum.

**Enduring Understanding 4.C:**
Quadratic functions can be used to model scenarios involving symmetry and a maximum or minimum.

Solutions

**Enduring Understanding 5.A:**
Solving an equation or inequality is a process of reasoning with algebraic rules to transform an equation or inequality into another equation or inequality with identical solutions.

**Enduring Understanding 5.B:**
Solutions to systems of equations or systems of inequalities simultaneously satisfy every equation or inequality in the system.

**Enduring Understanding 5.C:**
Solutions to equations and inequalities arising from contextual scenarios have meaning about the real-world situations from which they were modeled.
Instructional Units

Pre-AP Algebra 1 is organized by four units:

<table>
<thead>
<tr>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
<th>Unit 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Equations and Function</td>
<td>Systems of Linear Equations and Inequalities</td>
<td>Quadratic Functions</td>
<td>Exponent Properties and Exponential Functions</td>
</tr>
<tr>
<td>9 weeks</td>
<td>5 weeks</td>
<td>9 weeks</td>
<td>5 weeks</td>
</tr>
</tbody>
</table>

The following resources* are provided for each unit to support teachers and students:

**Lessons:** Lessons are designed to support the instructional shifts for the course and will therefore emphasize problems with authentic contexts and opportunities for mathematical modeling and argumentation. Lesson resources will vary in instructional length, but more robust lesson support is provided in Unit 1 for guidance on the instructional shift of mastery of linear equations and functions from procedural, conceptual, and application perspectives.

**Practice:** Recommendations for student practice are included within lessons, including connections to Khan Academy practice to support foundational math skills.

**Performance Task:** One performance task and scoring rubric for each unit.

**Assessments:** Two short objective assessments per unit, administered digitally to provide immediate performance feedback and score reporting.

*These resources constitute less than 50% of instructional time for the course, and they are intended to be used alongside current Algebra 1 textbooks and local district curriculum materials.
Unit Outlines

Unit 1: Linear Equations & Functions

Unit Overview
Students will explore all aspects of linear relationships: contextual problems that involve rate of change, lines in the coordinate plane, numerical patterns with a constant rate, and algebraic means of expressing a linear relationship between two quantities. Through this unit, students will develop an appreciation for the power and simplicity of linear functions as building blocks of all higher mathematics.

In this unit, students will build these key understandings:
- Linear relationships have a constant rate of change.
- Linear relationships can be expressed in an algebraic form, in a graphical form, in a tabular form, or in a contextual scenario.
- Linear functions are useful for modeling linear contextual situations and for making sense of some nonlinear situations.
- Linear models can be developed from pairs of related data and applied to make predictions in context.

Key Concepts
- Direct variation
- Slope and rate of change
- Linear functions
- Linear equations
- Scatterplots and lines of fit
- Linear inequalities

Unit 2: Systems of Linear Equations and Inequalities

Unit Overview
Students will explore systems of linear equations and inequalities, with special focus on the usefulness of modeling physical phenomena with a system. The goal of solving a system of equations in Algebra 1 is both to acquire an answer and for students to become strategic and efficient about choosing a method to solve the system.

In this unit, students will build these key understandings:
- The solution to a system of equations or inequalities is an ordered pair of numbers that satisfies all equations or inequalities.
- Systems of linear equations or inequalities are useful for modeling scenarios that involve resource limitations, goals, constraints, comparisons, and tolerances.

Key Concepts
- The solution to a system of equations
- Techniques for solving a system of linear equations
- Systems of inequalities
- Modeling with systems of equations and inequalities
Unit 3: Quadratic Functions

Unit Overview
Students get a thorough grounding in an important nonlinear function, quadratic functions. Students will understand that quadratic functions are formed by multiplying two linear expressions and therefore are not linear. Quadratic functions are useful for modeling phenomena that have a maximum or minimum value and exhibit symmetry such as objects in free fall.

In this unit, students will build these key understandings:
- A quadratic function is formed by the product of two linear expressions.
- The graph of a quadratic function is symmetric about the vertical line containing its maximum or minimum.
- Quadratic functions can be used to model scenarios involving symmetry and a maximum or minimum.
- A quadratic equation, \( ax^2 + bx + c = 0 \) where \( a \) is not zero, has at most two distinct solutions.

Key Concepts
- Modeling with quadratic functions
- Solving quadratic equations
- Algebraic forms of a quadratic function
- The graph of a parabola

Unit 4: Exponent Properties and Exponential Functions

Unit Overview
Students will explore exponent rules as an extension of both geometric sequences and the properties of multiplication and division for real numbers. Students should make sense of exponent rules and not simply memorize them without understanding how they arise. The unit culminates in an investigation of how exponential functions can be utilized to model physical phenomena that exhibits a constant ratio of growth. Exponential functions will be connected to linear functions because they have similar properties.

In this unit, students will build these key understandings:
- Exponents are a useful notation whose properties are derived from the properties of multiplication and division.
- Exponential functions can be used to model physical scenarios that involve constant multiplicative growth or decay.

Key Concepts
- Exponent rules and properties
- Roots of real numbers
- Exponential growth and decay
Finding the Constant Rate

Purpose

The aim of this lesson is to give students a clear and concise method to find the constant rate. Students should understand that an entire direct variation could be determined using just a single pair of numbers from the relationship. Two quantities are said to vary directly if one of them is a constant multiple of the other, equivalently if their quotient is constant. If quantities $x$ and $y$ vary directly, then they can be related by the equation $y = kx$ for some non-zero constant $k$. A portion of the class time in this lesson is devoted to student practice. Students will practice finding the constant rate, writing equations, calculating tables of values, and constructing a graph.

Lesson Elements

The lesson has three components:

- The first part is a review of direct variation and calculating the constant rate. Students have experienced several days of these kinds of problems and should not find it difficult to complete the task.

- The second part provides students with an opportunity to practice with direct variation, finding the constant rate, and using it to answer questions. What distinguishes this practice from more traditional kinds of rote skill-building is the use of realistic contexts throughout the assignment. Students practice with problems that involve interesting scenarios and thought-provoking questions. Most practice assignments provide students a chance to reflect on what they have learned.

- In the final part, students should complete a short formative assessment problem. This will provide the teacher with information to determine whether students understand direct variation and how to calculate the constant rate. Students who struggle with either of these skills may benefit from additional support resources available through Khan Academy.

Practice with Khan Academy

Build and Support Foundational Skills

Some students may need more support with direct variation. See these Khan Academy resources for these topics:

- Identifying Proportional Relationships
- Proportional Relationship Graphs
Finding the Constant Rate

Using the information on this sign, how can you determine how many feet are in 1 meter?

Can you also figure out how many meters are in 1 foot?

Determine how many feet is equal to 19 meters:

Determine how many meters is equal to 12 feet:
Finding the Constant Rate Practice

Problem 1:
Currency exchange is a direct variation. That means any currencies can be converted to any other currency using multiplication, if you know the exchange rate. The table below shows some exchange rates for several world currencies on June 5, 2017. The table is read from the left. For example, 1 United States dollar (USD) could be exchanged for 0.88872 euros (EUR) or 110.489 Japanese yen (JPY).

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>EUR</th>
<th>GBP</th>
<th>INR</th>
<th>AUD</th>
<th>CAD</th>
<th>ZAR</th>
<th>NZD</th>
<th>JPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 USD</td>
<td>1.00000</td>
<td>0.88872</td>
<td>0.77470</td>
<td>64.3456</td>
<td>1.33521</td>
<td>1.34741</td>
<td>12.7056</td>
<td>1.40021</td>
<td>110.489</td>
</tr>
<tr>
<td>1 EUR</td>
<td>1.12521</td>
<td>1.00000</td>
<td>0.87169</td>
<td>72.4022</td>
<td>1.50239</td>
<td>1.51612</td>
<td>14.2965</td>
<td>1.57553</td>
<td>124.323</td>
</tr>
<tr>
<td>1 GBP</td>
<td>1.29083</td>
<td>1.14719</td>
<td>1.00000</td>
<td>83.0592</td>
<td>1.72352</td>
<td>1.73928</td>
<td>16.4008</td>
<td>1.80743</td>
<td>142.622</td>
</tr>
<tr>
<td>1 CNY</td>
<td>0.14696</td>
<td>0.13061</td>
<td>0.11385</td>
<td>9.45637</td>
<td>0.19622</td>
<td>0.19802</td>
<td>1.86725</td>
<td>0.20578</td>
<td>16.2376</td>
</tr>
</tbody>
</table>

Mid-market rates: 2017-06-05 1832 UTC

a. What is the exchange rate for converting British pounds (GBP) to Australian dollars (AUD)?

b. A British resident is traveling in Australia. She goes to an ATM machine and withdraws 500 AUD. How many British pounds does the bank remove from her account back home?

c. Using the information in the table, determine the exchange rate for Canadian dollars (CAD) to Japanese yen (JPY).

d. A Japanese student wants to buy clothes from a Canadian store online. If the pants he wants cost 54 CAD, how much JPY is the student going to spend?

Problem 2:
To meet its production orders, the warehouse of Basic Office Supplies has to pack and ship 6 orders every hour. The warehouse runs 24 hours per day. If the warehouse reports that they have packed and shipped 288 orders by the end of the second day, are they meeting their goal of 6 orders every hour?
Problem 3:
A hurricane can produce heavy rain for a prolonged period of time. Suppose that Hurricane Amanda is expected to produce 12 hours of rainfall at a rate of 0.7 inches per hour. Construct a table of values to show how much rain would be accumulated over the span of the storm.

<table>
<thead>
<tr>
<th>Hour</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem 4:
Ronnie has an after-school job at an animal shelter. She notices that at one meal, 3 cats will eat 2 cans of food.

a. How much food does one cat eat?

b. Write an equation that Ronnie can use to determine how many cans of food to use per meal based on the number of cats in the shelter.

c. If there are 12 cats in the shelter, how many cans of food per meal should Ronnie expect to use to feed the cats?

Problem 5:
Every time Kylie hits the snooze button on her alarm, the alarm gets delayed for 9 minutes.

a. Is the relationship between the number of times Kylie hits the snooze button and the amount of time the alarm gets delayed an example of a direct variation? Explain your answer.

b. Construct a graph of the relationship between the number of times Kylie hits the snooze button and the amount of time the alarm gets delayed:
Finding the Constant Rate Formative Assessment

Randy wants to run a half marathon, which is 13.1 miles, next weekend.

- If she wants to run the race in 2 hours, what rate in minutes per mile should she try to maintain for the race?

- At the 10-mile mark, Randy realizes that her total running time is 90 minutes. Has Randy been keeping the pace (rate) that she intended? Explain your answer.
Sample Lesson

Slope as a Rate of Change

Purpose

Before learning the formula to calculate the slope of a line, students should explore rate of change in context. In this lesson students will interpret the rate of change as amount the dependent variables changes as the independent variable changes by 1. There are two important mathematical components to this lesson. The first is recognizing the proportional relationship between $\Delta y$ and $\Delta x$. Some students may be familiar with the notation $m = \frac{\Delta y}{\Delta x}$, but it is powerful to explore the equivalent version $\Delta y = m \cdot \Delta x$. The second component is a continuation of defining slope as the change in the dependent variable as the independent variable changes by 1. Often in scientific contexts, the rate of change of a linear relationship is stated this way.

Lesson Elements

The lesson should unfold in two parts.

- In the first part, students will engage with a contextual scenario involving a truck’s distance from the warehouse and the time traveled. The data from this problem is represented in a table of values, which will help students recall their work with arithmetic sequences and rate of change from previous lessons. Students can answer the questions individually or with a partner. When students have finished answering the questions, a whole-class discussion should make clear the connection between the additive relationship that each distance is 65 miles more than the previous one, and the multiplicative relationship between the number of hours driven and the distance traveled in that time.

- The second part of the lesson displays information from a car trip on a graph. This is different from the truck problem because the independent variable is the mile marker passed at a given time, not the distance traveled (although the distance traveled can be ascertained from the mile marker information). Because none of the mile markers provided have an increment of 1 hour, students will need to determine how to get the per hour rate. This portion of the lesson should conclude with a whole-class discussion summarizing the observation that the unit rate can be calculated by determining the change between values of the independent variable and dividing it by the corresponding change between the dependent variables.
**Slope as a Rate of Change**

A truck driver is beginning her haul across the country. The warehouse is 15 miles from the highway. Once she gets on the highway she maintains a constant rate for several hours. Complete the table of values for the sequence below that relates hours driven and distance from the warehouse.

<table>
<thead>
<tr>
<th>$h$, hours driven</th>
<th>$D$, Distance from warehouse</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>145</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

- By how much does the trucker’s distance from the warehouse increase every hour?

- What is the rate of change of the sequence?

- What is the truck’s speed?

- By how much will the distance from the warehouse increase in two hours? How do you know?

- By how much will the distance from the warehouse increase in three hours? How do you know?

- By how much will the distance from the warehouse increase in four hours? How do you know?
You are on a road trip that will take several hours on a single highway, and there is no traffic. To pass the time, you pay attention to the mile markers and how quickly they come. When you got on the highway, you entered next to mile marker 135. You kept track of the time, \( t \), that passed in hours and what mile marker, \( m \), you were passing on a graph. You noticed that the graph looked like a line.

- Is the slope of the line positive or negative? Explain the meaning of the slope in context.

- What is the slope of this line? How did you determine your answer?

- What does this slope mean in context?

- How many miles did you travel in 3 hours?

- If you were to travel for 8 hours, how many miles would that be total? What mile marker would you see?

- How often will you see a mile marker?
Sample Lesson

Revenue and Profit

Purpose

This lesson is designed to span several class periods and provide students with an opportunity to model an economic phenomenon with a quadratic function. Students will leverage their knowledge of linear functions to build components of the model and will use a quadratic function to maximize the profit from selling pies at a bake sale. Students will also learn the standard from of a quadratic function.

It is important not to rush through the modeling process here. Students need to see that real mathematical modeling involves making and testing conjectures and revising formulas or adjusting assumptions when appropriate. The numbers in this scenario were chosen to be for relative ease of computation so students can invest substantial cognitive effort in understanding the process of creating, using, and interpreting a mathematical model.

Lesson Elements

The model will unfold in this order: first the cost function, then the price function, then the revenue function, and finally the profit function. Students should work in small groups on each part of the problem, then share their work with the whole class. The questions included in the student material on the subsequent pages can be used to guide a whole class discussion.
Revenue and Profit

Every year, the Math Club at your school sells pies on Pi Day as a fundraiser. Since you have done this for several years, you have data to use in setting the price. (When you made 160 pies, you sold all the pies at $6 per pie. When you made 80 pies, you sold all the pies for $10 each.) Your principal has allowed you to use the school cafeteria kitchen for a one-time, nonrefundable fee of $300, and the ingredients for each pie cost $2.

How many pies should you make to maximize profit? How much should you sell the pies for? How much money will you make?

Part 1: Cost function
- What do we know about the costs for making pies?
- What do these quantities mean?
- What kinds of functions have constant rates?
- How do you write a linear function with a constant rate and an initial value?

Part 2: Price function
- What information are we given about the price of the pies?
- Which of the quantities in the problem is the input? Which is the output?
- How can we represent the information as two ordered pairs of numbers?

Part 3: Revenue function
- If we were to sell 15 pies at $6 each, how much revenue would we make?
- If we were to sell 1 pie at $6 each, how much revenue would we make?
- If we were to sell 378 pies at $6 each, how much revenue would we make?
- If we were to sell \( x \) pies at $6 each, how much revenue would we make?
- How do you calculate revenue if you know the price per item and the number of items you sell?
- If we were to sell \( x \) pies at \( E \) dollars each, how should we calculate our revenue?
- Create a graph of the revenue function.
- Which part of the graph will indicate the maximum revenue?
- Can you determine the coordinates of that point?
- What does that ordered pair of numbers mean in the context of the problem?
- Is our goal to maximize revenue?
Part 4: Profit function

- What is the goal of the problem?
- How do we calculate the profit?
- What is the profit if we sell 0 pies?
- What does a negative profit mean?
- Looking at the equation, do you predict that this will be a line? Another shape? Why or why not?
- Which part of the graph will reveal the maximum profit?
- What are the coordinates of that point?
- What does it mean in the context of the problem?
- Is the number of pies we should sell to maximize revenue the same as the number of pies to maximize profit? Why do you think this is?
The Weight-Lifting Program

Darius wants to play football in the fall, and his coach wants all players to complete a workout regimen for the summer. He needs to consistently increase the number of pounds that he bench presses. The coach told Darius to add only 10 pounds to the bar each week. Darius is relatively new to weight lifting, so he decides to start his bench press routine using only the bar. The bench press bar weighs 45 pounds.

a. Write an equation that relates the total number of pounds, \( p \), that Darius bench presses to the number of weeks he has been working out, \( w \). Explain how you know that this equation represents a linear relationship.

\[ p = 45 + 10w \]

b. If Darius’ summer is 10 weeks long and he follows his coach’s advice all summer, how many pounds should he be able to bench press by the end of the summer?

\[ p = 45 + 10(10) = 45 + 100 = 145 \text{ pounds} \]

c. Darius’s friend Juan is also working out all summer. Juan keeps a workout log of the total weight he has been bench pressing, but some weeks he forgot to write down the amount.

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>95</td>
<td>100</td>
<td>110</td>
<td>115</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If Juan followed a consistent pattern for the entire summer, how much weight would he have lifted in weeks 4 and 7? Explain your answer.

\[ p = 45 + 10(4) = 45 + 40 = 85 \text{ pounds} \]

\[ p = 45 + 10(7) = 45 + 70 = 115 \text{ pounds} \]

d. Suppose that Juan and Darius began working out on the same week. Who will be lifting more weight by the end of the 10-week summer? Show all your work and explain your answer.

\[ (p_{Juan})_{10} = 45 + 10(10) = 45 + 100 = 145 \text{ pounds} \]

\[ (p_{Darius})_{10} = 45 + 10(10) = 45 + 100 = 145 \text{ pounds} \]

Both Darius and Juan will be lifting 145 pounds by the end of the 10-week summer.
The Weight-Lifting Program

Possible Points | Solution/Answer
--- | ---
1 point: Correct equation | \( p = 45 + 10(w - 1) \)
1 point: Correct explanation | It is a linear relationship because there is a constant rate of change (10).

2 points: Correct answer using part (a) equation OR
1 point: Correct answer by constructing a table of values to count the amount of weight per week

When \( w = 10 \), \( p = 45 + 10(10 - 1) = 135 \) pounds

<table>
<thead>
<tr>
<th>( w )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>45</td>
<td>55</td>
<td>65</td>
<td>75</td>
<td>85</td>
<td>95</td>
<td>105</td>
<td>115</td>
<td>125</td>
<td>135</td>
</tr>
</tbody>
</table>

1 point: Correct answer for both weeks 4 and 7

Rate of change is 5 pounds per week, so in week 4 he lifts 105 pounds and in week 7 he lifts 120 pounds.

2 points: Correct value for Juan’s weight by constructing an equation or by continuing Juan’s table of values to week 10

Juan: \( p = 90 + 5(w - 1) \)

In week 10: \( p = 90 + 5(10 - 1) = 135 \) pounds

<table>
<thead>
<tr>
<th>( w )</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>125</td>
<td>130</td>
<td>135</td>
</tr>
</tbody>
</table>

1 point: Correct comparison (They will have the same weight.)

They will be lifting the same amount of weight by the end of the 10-week summer.
The information included in this sampler is still subject to change, as Pre-AP courses are still being developed with teacher feedback. Pre-AP course materials, including the course framework, lessons, and unit assessments will be finalized in spring 2018. Pre-AP courses will launch in fall 2018.